CAUSAL UNIVARIATE SPATIAL-TEMPORAL AUTOREGRESSIVE MOVING AVERAGES (STARMA) MODELLING OF TARGET INFORMATION TO GENERATE TASKING OF A WORLD-WIDE SENSOR SYSTEM

THESIS

Kelly A Greene
Second Lieutenant, USAF

AFIT/GOR/ENS/92M-12



DEPARTMENT OF THE AIR FORCE
AIR UNIVERSITY
# AIR FORCE INSTITUTE OF TECHNOLOGY

Wright-Patterson Air Force Base, Ohio

AFIT/GOR/ENS/92M-12



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| COMMITTEE: | NAME/DEPARTMENT | SIGNATURE |
|---|---|---|
| Advisor | Dr. Yupo Chan/ENS | Yupo Chan |
| Reader | Dr. Edward F. Mykytka/ENS | Edward F. Mykytka |

AFIT/GOR/ENS/92M-12

# CAUSAL UNIVARIATE SPATIAL-TEMPORAL AUTOREGRESSIVE MOVING AVERAGE (STARMA) MODELLING OF TARGET INFORMATION TO GENERATE TASKING OF A WORLD-WIDE SENSOR SYSTEM

THESIS

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Kelly A Greene, B.S.

Second Lieutenant, USAF

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## *Preface*

The purpose of this research was to establish a methodology using a univariate causal STARMA model for forecasting the relative probability of an event occurring in a geographical location during a time block of the day. These relative probabilities are used as input for a tasking model that assigns scarce sensor resources so as to optimize the detection of these events. The model created is a univariate causal STARMA model in that it only produces forecasts for one of the twenty-two given geographical regions. The model was created to provide forecasts for one event type occurring at geographical region 11 and appears to provide good forecasts. Future research may show that the univariate causal STARMA methodology is a feasible approach to generate forecasts for the other event types and other geographical regions.

I owe many thanks to Dr. Yupo Chan, my thesis advisor, and to Dr. Edward F. Mykytka, my reader. I also thank Dr. Alfred B. Marsh III, Christopher Dearing, and Ronald C. Adamowicz from the Department of Defense for sponsoring my research. A word of thanks is also in order for Lt. Col. James Robinson (USAF, Retired) whose technical assistance with STARMA modelling was invaluable. Finally, I thank my family and friends for their love and support during the past six months.

Kelly A Greene

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*Abstract*

The Department of Defense employs a world-wide sensor system to detect certain "events" of interest. This system is resource limited and therefore incapable of providing coverage everywhere. The purpose of this research was to establish a methodolgy using a univariate causal STARMA model for forecasting the relative probability of an event occurring in a geographical location during a time block of the day. These relative probabilities are used as input for a tasking model that assigns the scarce sensor resources so as to optimize the detection of these events.

Given that an event occurs, where it occurs is controlled by some plan or doctrine which is unknown, but for which a hypothesized model exists. As a result, there exists an analytical data base that consists of forecasted analytical relative probabilities from the hypothesized model. There also exists a historical data base that consists of the relative probabilities observed by the world-wide sensor system. The causal STARMA model created uses the information from both data bases in an attempt to create forecasts that are better than that of the analytical model.

The STARMA model is appropriate for forecasting the relative probabilities because a definte temporal relationship and a definite spatial relationship exists in the data bases. What has happened in the past is a good indication of what may happen in the future. Also, a strong relationship between relative probabilities of the twenty-two geographical regions exists.

The model created is a univariate causal STARMA model in that it only produces forecasts for one of the twenty-two given geographical regions. A STARMA model would produce forecasts for all twenty-two geographical regions simultaneously. The research was limited to univariate STARMA modelling due to lack of STARMA software (at least locally). A causal univariate STARMA model was created to provide forecasts for one event type occurring at geographical region 11 and appears to provide good forecasts. The model is both correlative and causal. The model is correlative in

that it uses temporal and spatial correlations to develop the forecasts. The model is also causal in that it employs the predictions from the analytical model.

Future research may show that the univariate causal STARMA methodology is a feasible approach to generate forecasts for the other event types and other geographical regions.

CAUSAL UNIVARIATE SPATIAL-TEMPORAL AUTOREGRESSIVE MOVING AVERAGE (STARMA) MODELLING OF TARGET INFORMATION TO GENERATE TASKING OF A WORLD-WIDE SENSOR SYSTEM

# *I. Introduction*

## *1.1 Introduction to the Problem*

The United States Department of Defense employs a world-wide sensor system to detect certain "events" of interest. This system is resource limited and therefore incapable of providing coverage everywhere (14:1).

A tasking model was designed to allocate the scarce sensor resources so as to optimize the detection of these events. The success of the tasking algorithm depends upon the input of good estimates for the conditional probabilities $P_{ijk}$, aggregated monthly, of event type $i$ occurring in area $j$ at time $k$ given that an event type $i$ occurred at time $k$. It should be noted that $i$ ($i = 1, 2, 3$) indexes the event type, $j$ ($j = 1, 2, ..., 22$) indexes the geographical region, and $k$ ($k = 1, 2, ..., 12$) indexes the time of day (14:1). The time of day $k$ is divided into two hour time blocks. For example, $k = 1$ may represent the two hour time block from 0600 hours to 0800 hours , $k = 2$ may represent the two hour time block from 0800 hours to 1000 hours, et cetera.

These conditional probabilities $P_{ijk}$ are expressed as the following product:

$$P_{ijk} = p_{ijk} \cdot q_{ik} \tag{1}$$

where $P_{ijk}$ is the conditional probability that event type $i$ occurs at geographical region $j$ at time of day $k$ given that event type $i$ occurs at time of day $k$,

$p_{ijk}$ is the relative probability that event type $i$ occurs at geographical

region $j$ at time of day $k$, and

$q_{ik}$ is the probability that an event type $i$ occurs at time $k$.

The probability $p_{ijk}$ is a relative probability because the $p_{ijk}$'s over a given $j$ sum to 1.0. It is assumed that $q_{ik} = 1.0$ and thus, the present concern is for accurately estimating the $p_{ijk}$ factor (14:1).

Given that an event occurs, where it occurs is controlled by some plan or doctrine which is unknown, but for which an hypothesized model exists. This model generates analytical predictions $\breve{p}_{ijk}$ of the desired probability $p_{ijk}$, aggregated monthly. The purpose of this research is not to investigate the analytical model and thus, it is assumed that the analytical model is adequate. The accuracy of these predictions $\breve{p}_{ijk}$ depends upon the strength of the hypothesized model (14:1).

A historical data base exists that consists of the relative frequencies $X_{ijk}$ of occurrences observed by the world-wide sensor from January of 1985 through July of 1991. Each relative frequency $X_{ijk}$ is aggregated over each month, resulting in a total of 79 months of relative aggregated frequencies $X_{ijk}$ for each event type $i$, each geographical region $j$, and each time block of the day $k$. The historical data base contains a total of 62,568 (3 x 22 x 12 x 79 = 62,568) relative frequencies $X_{ijk}$. A combination of the historical data $X_{ijk}$ and the corresponding analytical prediction data $\breve{p}_{ijk}$ should result in a prediction $\hat{p}_{ijk}$ more accurate than that of the analytical model (14:1). The accuracy of the predictions $\hat{p}_{ijk}$ depends upon the adequacy of the historical data base $X_{ijk}$ and the analytical model estimates $\breve{p}_{ijk}$, as well as the correlation of future regional event occurrences to past regional event occurrences (14:1).

### 1.2 Main Objective

The main objective of this thesis is to establish a methodology for forecasting the relative monthly probability $\hat{p}_{ijk}$ one month into the future in order to task for a world-wide sensor system by combining historical data $X_{ijk}$ with estimates $\breve{p}_{ijk}$ from an existing analytical prediction model.

To scale the problem down, the focus of this research will be limited to event type 2 of the three events types and to time block 1 of the twelve time blocks. Thus, the forecasts will be built using the historical monthly relative frequencies and the analytical monthly predictions for event type 2 and time block 1 for all 22 geographical regions.

### 1.3 Secondary Objective

The secondary objective of this thesis is to present and summarize alternative methods for modelling the relative monthly probability $\hat{p}_{ijk}$ along with a comparison of these alternative methods. Policy or doctrinal changes and other external events can trigger an abrupt or gradual change in the mean or trend of a time series. When changes happen abruptly, a Box-Jenkins model class may take several periods to "catch up" with the effects of the external phenomena. In order to produce a better model, one of the techniques of intervention analysis can be applied. The intervention analysis methods discussed include classical intervention analysis, simple exponential smoothing, adaptive response rate exponentially smoothing, the Kalman filter, and multiattirbute utility theory.

# *II. Literature Search and Review*

## *2.1 Introduction*

The following paragraphs will summarize literature pertinent to this research. The literature summary covers the methodologies that will be used to forecast the probability $\hat{p}_{ijk}$ of an event $i$ occurring at geographical region $j$ at time of day $k$ one period into the future. Specifically, the discussion covers the following topics: model classes; the ARMA model class; the STARMA model class; the ARMA model class versus the STARMA model class; and the model building procedures of the Box-Jenkins method.

## *2.2 Model Classes*

There are many model classes that describe and forecast a set of random variables distributed over time and space. Four flexible, empirical model classes are the Autoregressive Moving Average (ARMA) model class, the Vector Autoregressive Moving Average (VARMA) model class, the Transfer Function model class, and the STARMA model class. A modeler will commonly build all four of these model classes using the Box-Jenkins method (20:35). The Box-Jenkins method is an iterative approach with four stages: identification, estimation, diagnostic checking, and forecasting, as will be discussed in Section 2.6 (12).

The ARMA model class is univariate and is only applicable to a single time series of data; the ARMA model class can only deal with past observations at a particular point in space (20:35). Special subclasses of the ARMA model class are the Autoregressive (AR) model and the Moving Average (MA) model. A special case of the ARMA model is the Seasonal Autoregressive Moving Average (SARMA) model that occurs when seasonality is present in the time series.

The VARMA model class allows for the modeling of multiple time series (24:802). The VARMA model class can describe $N$ time series of the same variable at different points in space but can not account for the interrelationships between the $N$ time series (20:35).

The Transfer Function model class allows for the modeling of multiple time series, also. The Transfer Function model class can describe $N$ time series consisting of different variables at the same point in space (4:74). The Transfer Function model class will account for the interrelationships between the $N$ time series, but cannot account for spatial differences (4:74).

The STARMA model class is useful in describing time series of spatially located data (11:401).

> Processes amenable to modeling via this model class are characterized by a single random variable observation at $N$ fixed sites in space, wherein dependencies between the $N$ time series are systematically related to the relative physical location of the sites. (18:255)

### *2.3 The ARMA Model Class*

Let $x(t)$ be an observation at time t. The ARMA model class can be written as:

$$x(t) = \xi + \sum_{k=1}^{p} \phi_k x(t-k) - \sum_{k=1}^{q} \theta_k \epsilon(t-k) + \epsilon(t) \tag{2}$$

where $p$ is the autoregressive order,

$q$ is the moving average order,

$\phi_k$ is the autoregressive parameter of order $k$,

$\theta_k$ is the moving average parameter of order $k$

$\xi$ is the constant term, and

$\epsilon(t)$ are error terms (16:253).

The common designation of an ARMA model is:

$$ARMA(p,q) \tag{3}$$

*2.3.1 Two Special Subclasses of the ARMA Model Class.* Two special subclasses of the ARMA model class are the Autoregressive (AR) subclass and the Moving Average (MA) subclass. The ARMA model class is a combination of the AR model subclass and the MA model subclass. An AR model subclass is appropriate when there are no significant moving average parameters in the model (11:401). This occurs when the moving average order $q = 0$ and the only significant parameters are autoregressive (11:401). The common designation of an AR model is:

$$AR(p) \tag{4}$$

The MA model subclass contains no significant autoregressive parameters (11:401-402). This occurs when the autoregressive order $p = 0$ and the only significant MA parameters are moving average (11:401-402). The common designation of a MA model is:

$$MA(q) \tag{5}$$

*2.3.2 The Seasonal ARMA Model Class.* It is possible to have a Seasonal Autoregressive Moving Average (SARMA) model.Seasonlity is characterized by a periodic movement in the data that almost always repeats itself. The season is defined as the length of time over which the periodic movement occurs. Typically, the length of a season is one year. However, it is possible to have a season that repeats itself every six months or every two years. The common designation of a SARMA model is:

$$SARMA(p,q)_{sar,sma} \tag{6}$$

where $sar$ is the seasonal autoregressive season length and

$sma$ is the seasonal moving average season length.

The common designation of a Seasonal Autoregressive (SAR) model is:

$$SAR(p)_{sar} \tag{7}$$

The common designation of a Seasonal Moving Average (SMA) model is:

$$SMA(q)_{sma} \tag{8}$$

*2.3.3 Assumption of the ARMA Model Class.* The ARMA model class assumes that $\epsilon(t)$, the error terms, are independent random normal error terms with zero mean and constant variance (16:242-243).

### *2.4 The STARMA Model Class*

Let $x(t)$ represent a $N$ x 1 vector of observations at time $t$ at $N$ locations. The STARMA model family can be written as:

$$x(t) = \xi + \sum_{k=1}^{p}\sum_{l=0}^{\lambda_k} \phi_{kl} W_l x(t-k) - \sum_{k=1}^{q}\sum_{l=0}^{m_k} \theta_{kl} W_l \epsilon(t-k) + \epsilon(t) \tag{9}$$

where $\xi$ is the constant term,

$p$ is the temporal autoregressive order,

$q$ is the temporal moving average order,

$\lambda_k$ is the spatial order of the $k$th autoregressive term,

$m_k$ is the spatial order of the $k$th moving average term,

$\phi_{kl}$ is the autoregressive parameter of temporal order $k$ and spatial order $l$,

$\theta_{kl}$ is the moving average parameter of temporal order $k$ and spatial order $l$

$W_l$ is the $N$ x $N$ weight matrix of spatial order $l$, and

$\epsilon(t)$ are error terms (18:256).

The common designation of the STARMA model is:

$$STARMA(p_{\lambda_1,\lambda_2,\cdots,\lambda_p}, q_{\lambda_1,\lambda_2,\cdots,\lambda_q}) \tag{10}$$

The $W_l$ weight matrix has non-zero elements $W_{ijk}$ only for those pairs of sites $i$ and $j$ that are $l$th-order neighbors (20:36). It is assumed that $W_l$ is the identity matrix $I$ when $l = 0$. First-order neighbors are closer than second-order neighbors that are closer than third-order neighbors, et cetera (18:256). The weights allow for a general specification of both a weighting scheme and a hierarchical ordering of spatial neighbors (21:23). Each row of the weight matrix must sum to one (21:26). The specification of the weights should reflect some physical property of the modelled system (20:36). Weights can be used to specify such things as the distance between points in space, common boundaries between points in space, or natural barriers such as rivers between points in space (20:36).

*2.4.1 Two Special Subclasses of the STARMA Model Class.* Two special subclasses of the STARMA model class are the Spatial-Temporal Autoregressive (STAR) subclass and the Spatial-Temporal Moving Average (STMA) subclass. The STARMA model class is a combination of the STAR model subclass and the STMA model subclass. A STAR model subclass is appropriate when there are no significant moving average parameters in the model (11:401). This occurs when the temporal moving average order $q = 0$ and the only significant parameters are autoregressive (11:401). The common designation of a STAR model is:

$$STAR(q_{\lambda_1,\lambda_2,\ldots,\lambda_q}) \tag{11}$$

The STMA model subclass contains no significant autoregressive parameters (11:401-402). This occurs when the temporal autoregressive order $p = 0$ and the only significant STMA parameters are moving average (11:401-402). The common designation of a STMA Model is:

$$STMA(q_{\lambda_1,\lambda_2,\ldots,\lambda_q}) \tag{12}$$

*2.4.2 The Seasonal STARMA Model Class.* It is possible to have a Seasonal Spatial-Temporal Autoregressive Moving Average (SSTARMA) model. The common designation for a seasonal STARMA model is:

$$SSTARMA(p_{\lambda_1,\lambda_2,\ldots,\lambda_p}, q_{\lambda_1,\lambda_2,\ldots,\lambda_q})_{sar,sma} \tag{13}$$

where $sar$ is the seasonal autoregressive season length and

$sma$ is the seasonal moving average season length.

The common designation of a Seasonal Spatial-Temporal Autoregressive (SSTAR) model is:

$$SSTAR(p_{\lambda_1,\lambda_2,\ldots,\lambda_p})_{sar} \tag{14}$$

The common designation of the Seasonal Spatial-Temporal Moving Average (SSTMA) Model is:

$$SSTMA(q_{\lambda_1,\lambda_2,\ldots,\lambda_q})_{sma} \tag{15}$$

*2.4.3 Assumption of the STARMA Class Model.* To simplify the model building procedure in the STARMA model class, one assumption is made. The assumption, known as the sphericity assumption, says that $\epsilon(t)$, the error terms, are independent random normal error terms with zero mean and constant variance (11:402). This assumption is called the sphericity assumption because it assumes that the contours of constant density of the error terms are spheres (11:402).

### *2.5 STARMA versus ARMA*

The elements of a STARMA model include both time and spatial dependence lagged in time and space to allow for temporal and spatial correlation (15:96). Temporal correlation is a pattern in the time dimension. Spatial correlation is a pattern in the space dimension. STARMA models reflect the ideas " ... that the recent past exerts more influence than the distant past ... and that near sites exert more influence on each other than distant ones" (18:255).STARMA can be applied to a wide range of problems with a spatial-temporal data structure. Past applications of the STARMA model class include river flow (17), population diffusion (4), hotel demand (18), and the spread of disease (22).

The STARMA model class is a forecasting tool that can predict future observations based on past observations at $N$ geographical locations and a given hierarchical spatial weighting. The ARMA model class also can represent and forecast $N$ time series each at a different site; however, each site has a different ARMA model. Though the ARMA approach will require construction of $N$ separate models, both model building approaches require about the same amount of effort (18:258). The STARMA model class provides many advantages over the ARMA model class when used as a forecasting tool.

In a study comparing the STARMA forecasting approach to that of ARMA, "the STARMA approach clearly produced better forecasts" (18:267). Average forecast errors using the STARMA representation are generally smaller than that of the ARMA representation (18:267). Pfeifer and Bodily credit the autoregressive spatial terms in the STARMA model as one of the reasons that STARMA produces better forecasts (18:270). They also believe that the STARMA model produces better results because it is a simultaneous estimation procedure that incorporates the correlation between sites (18:270). The STARMA model may produce better results because it is a simpler model that has fewer parameters than the $N$ ARMA models (18:270).

### 2.6 *Model Building Procedure of the Box-Jenkins method*

The Box-Jenkins method is a iterative approach that consists of four stages:

1. Identification.
2. Estimation.
3. Diagnostic Checking.
4. Forecasting.

If any stage is unsuccessfully completed, the procedure returns immediately to the identification stage.

*2.6.1 Stage One: Identification.* The purpose of the identification stage is to employ statistical procedures to specify tentatively which STARMA classes are appropriate for the data (24:805). The two statistics used to identify potential STARMA classes are the spatial-temporal autocorrelations and the spatial-temporal partial autocorrelations at temporal lag $l$ and spatial lag $k$ at time lag $s$ (19:119). The autocorrelations measure the relationship, or how much spatial-temporal interdependence exists, between data points of the $N$ time series (12). The modeler uses these autocorrelations to determine if the data is stationary. Statistically, a time series is stationary if the joint distribution of any sequence of observations $x(t+1)$, $x(t+2)$,..., $x(t+n)$ does not depend upon $t$ (5:26). In other words, the mean of the time series $E[x(t)]$, the variance of the time series $\sigma^2(t)$, and the lag $k$ autocorrelations of the time series $Corr[x(t), x(t+k)]$ do not change with $t$ (5:26,28). The joint distribution of any sequence of observations in a non-stationary time series does depend on $t$ and thus, there are several types of non-stationary data (5:26). An example of non-stationary data behaves as if the time series does not have a constant mean and a constant variance. In other words, the observations in any local segment of time look like the observations in any other segment, but the means and variances may differ significantly.

If the data is non-stationary in the mean, a technique called "differencing" can be applied. The differencing technique is discussed in more detail in Chapter 3. There are two types of time series that are non-stationary in the mean. The first type of data that is non-stationary in the mean behaves as if the time series does not have a constant mean (16:255). In other words, the observations in any local segment of time look like the observations in any other segment, but the means of the two local segments being compared differ significantly (16:255). The second type of data that is non-stationary in the mean behaves as if the time series does not have a constant mean and a constant slope (16:255). In other words, the observations in any local segment of time look like the observations in any other segment, but the means and slopes of the two local segments being compared differ significantly (16:255). For example, the time series may exhibit a linear trend or gradual shift in the data causing non-stationarity in the mean. As another example, the time series may exhibit a step change resulting in non-stationarity in the mean. The second type of non-stationarity in the mean can also result from several phenomena occurring in the data. For example, a non-linear trend or gradual shift in the slope can cause non-stationarity in the time series (16:255). A non-stationary time series that can be reduced to a stationary time series through differencing is a homogeneous non-stationary time series (16:257).If the data is stationary, the modeler can identify a potential subclass and the orders of the model.

The modeler identifies the subclass by examining the autocorrelations and the partial autocorrelations. A STAR process exhibits space-time partial autocorrelations that go to zero after $p$ lags in time and $\lambda_p$ lags in space (19:120). The space-time autocorrelations of a STMA process will go to zero after $q$ lags in time and $m_q$ lags in space (19:120). The modeler would further investigate a STARMA model class if both the autocorrelations and the partial autocorrelations exponentially go to zero.

Once the modeler identifies a model subclass, the temporal autoregressive order $p$, the temporal moving average order $q$, the spatial autoregressive order $\lambda_p$, and the spatial moving averages order $m_k$ are identified. If the modeler identifies a STAR model subclass, the only temporal order to determine is $p$ and the only spatial order to determine is $\lambda_p$. $p$ is identified to be equal to the number of temporal lags where the space-time partial autocorrelations are significantly different from zero. $\lambda_p$ is identified to be equal to the number of spatial lags where the space-time partial autocorrelations are significantly different from zero. If the modeler projects a STMA process, $q$ is the only temporal order to identify and $m_q$ is the only spatial order to determine. $q$ is identified to be equal to the number of temporal lags where the space-time autocorrelations are significantly different from zero. $m_q$ is identified to be equal to the number of spatial lags where the space-time autocorrelations are significantly different from zero. If the modeler identifies a STARMA model class, the temporal orders $p$ and $q$ of a STARMA model class are identified in the same fashion that $p$ and $q$ were identified for a STAR model subclass and a STMA model subclass, respectively (19:120).The spatial orders $\lambda_p$ and $m_q$ of a STARMA model class are identified in the same fashion that $\lambda_p$ and $m_q$ were identified for a STAR model subclass and a STMA model subclass, respectively (19:120).

The identification of a candidate model and its orders is never easy because the autocorrelations and the partial autocorrelations are only estimates. Every data set contains some noise or error component. A pure STAR, STMA, or STARMA process only exists in theory. It is possible for a data set to exhibit characteristics of many different model specifications. For example, a data set can exhibit characteristics of an STAR process but its space-time autocorrelations and space-time partial autocorrelations may exhibit the characteristics of a STARMA process. Model building is both an art and a science and will require the judgement of the modeler to choose the best candidate model and its orders.

*2.6.2 Stage Two: Estimation.* After choosing the best potential candidate model and its orders, the modeler estimates the autoregressive parameters $\phi_{kl}$ and the moving average parameters $\theta_{kl}$ (24:809). The efficient estimates of both $\phi_{kl}$ and $\theta_{kl}$ are maximum likelihood estimators (24:809). Calculating the maximum likelihood estimators of the parameters is not an easy task and therefore, the modeler usually approximates the parameters using a conditional likelihood function that minimizes the conditional sum of squares (20:41-42). Estimating the STAR parameters then is easy since the conditional likelihood function is also a least square estimate; the modeler can simply estimate the STAR model parameters using a linear regression (20:42). The estimation of the STMA and STARMA model parameters is not so effortless due to non-linearity. The modeler can employ various non-linear optimization techniques, such as gradient methods or linearization, to calculate the STMA and STARMA model parameters (20:42).

At this point, the modeler has a candidate representation of the data with estimated parameters. If necessary, the modeler can calculate confidence intervals around the parameters and the constant variance term.

*2.6.3 Stage Three: Diagnostic Checking.* The objective of diagnostic checking is to verify that the selected model is adequate. An adequate model sufficiently describes and sufficiently represents the data. The selected model must pass two tests to be an adequate model (18:257).

The first test examines the statistical significance of the model parameters (18:257). The model parameter of the highest order for $p$, $q$, $\lambda_p$, and $m_k$ must be significantly different from zero. If any parameter for its respective highest order is insignificant, then the modeler will return to the identification stage.

The second test verifies that the model residuals are white noise which, in turn, ensures that the model does not violate the sphericity assumption (20:43). If the space-time autocorrelations and the space-time partial autocorrelations of the residuals are all close to zero, then the model residuals are random. The analysis of the space-time autocorrelations "... guards against model mis-specification and searches for directions of improvement" (24:809). If a scatter plot of the residuals shows no patterns, then it can be concluded that the model residuals are from a random process. If the selected model fails the second test, the modeler will return to the identification stage to represent the residuals as a separate STARMA model class and combine it with the original model (20:43).

After passing the diagnostic check, the model is adequate to describe and represent the $N$ time series.

*2.6.4 Stage Four: Forecasting.* The first step in the forecasting stage is to select the best and most parsimonious model. There are two criterion used to select the best and most parsimonious model. The first criterion is the fraction of the variance described by the model, adjusted for the degrees of freedom, $\bar{R}^2$. The second criterion is the sum of the squared residuals, SSR. The best and most parsimonious model is that model which uses the smallest number of parameters necessary to adequately describe and represent the $N$ time series such that the $\bar{R}^2$ is maximized and the SSR is minimized. It is best to place more weight on the $\bar{R}^2$ value over the SSR value because the $\bar{R}^2$ value accounts for the number of parameters in the model by adjusting for the number of degrees of freedom. The $\bar{R}^2$ value can actually get worse in some cases as the number of parameters is increases, whereas the SSR value can only get better as the number of parameters increases. Once the best and most parsimonious model is selected, the model can be used to predict the $N$ time series.

### *2.7 Summary*

There are many empirical model classes that describe and forecast time series. The characteristics of the data drive the type of model class to use. When the data consists of time series at $N$ points in space that are spatially and temporally correlated, a STARMA model class will produce the best representation and description of the inherent, underlying processes.

There are four major iterative steps in building a STARMA model that can predict future observations: identification, estimation, diagnostic checking, and forecasting. A model that passes the diagnostic checking stage is the most adequate model. A model that passes the diagnostic checking and minimizes the temporal and spatial orders is the best and most parsimonious model to forecast future events.

# III. Methodology

## 3.1 Introduction

Due to the lack of STARMA software (at least locally), a full STARMA model on the twenty-two geographical regions will not be developed. Instead, a univariate STARMA model on one of the geographical regions will be developed. The main difference between a STARMA model and a univariate STARMA model is that a STARMA model would develop forecasts for all twenty-two geographical regions simultaneously whereas the univariate STARMA model develops forecasts for one geographical region, which is called the target region. The univariate STARMA model develops its forecasts using the temporal correlations in the target region time series and the spatial correlations between the target region time series and the time series of the other twenty-one regions. The model developed for the target region is appropriate for producing forecasts for the target region only. The univariate STARMA model is not meant to be a development for the full STARMA model. The univariate STARMA model uses the general approach of the STARMA model to produce spatial-temporal forecasts for the target region.

The methodology for developing a univariate STARMA model can be broken into seven steps:

1. Data analysis.
2. Autocorrelation Analysis.
3. Determination of Target Region, Neighbors, and Spatial Weights.
4. Identification.
5. Estimation.
6. Diagnostic Checking.
7. Forecasting.

This chapter explains the methodology of the nine steps used to develop a univariate STARMA model.

### 3.2 Step 1: Data Analysis

Data analysis is the first step. The historical data base consist of time series data of relative frequencies of occurrences, aggregated monthly. The analytical prediction model estimate data base consist of time series data of probabilities, aggregated monthly. Time series data is data collected over a certain amount of time. A relative monthly frequency $X_{ijk}$ in the historical data base represents the observed frequency that an event of type $i$ ($i = 1, 2, 3$) occurred in area $j$ ($j = 1, 2, ..., 22$) at time of day $k$ ($k = 1, 2, ..., 12$). A probability in the analytical prediction model estimate data base represents the predicted monthly probability $\breve{p}_{ijk}$ that an event of type $i$ ($i = 1, 2, 3$) occurs in area $j$ ($j = 1, 2, ..., 22$) at time of day $k$ ($k = 1, 2, ..., 12$). It should be noted that the analytical data base does not consists of *relative* probabilities. The historical data base includes all relative monthly frequencies from January 1985 through July 1991 resulting in a total of 62,568 (3 x 22 x 12 x 79 = 62,568) relative monthly frequencies. The analytical data base includes all monthly probabilities from January, 1985, through July, 1991, resulting in a total of 62,568 monthly probabilities.

The problem was scaled down to only include event type 2 and time of day 1. This reduced the total number of relative historical monthly frequencies to 1,738 (22 x 79 = 1,738) and the total number of analytical monthly probabilities to 1,738.

Data analysis begins with generating three-dimensional plots of the historical data for all geographical regions over time (January 1985 through July 1991) in order to look for possible patterns in the data. Version 3.0 of the software package *GNUPLOT* is used to generate the plots. *GNUPLOT* is an interactive plotting program. Examination of plots of the frequency distributions aggregated over each year may reveal a trend pattern, a gradual shift in the frequency distribution. Examination of plots of the frequency distributions aggregated over each of the four seasons may reveal a pattern of seasonality, a periodic movement in the data that almost always repeats itself every twelve months.

Experts familiar with the historical data base state that theoretically speaking, the historical data base should be trendless. The experts also state that several cycles or seasons are theoretically expected in the historical data. A cycle of 24 hours, a season of 12 months, and a cycle of 11 years is expected (2). Since the problem was scaled down to time block 1, the 24 hour cycle does not need to be modelled. Since the historical and analytical databases contain approximately six and one half years of data, there are not enough observations to model the 11 year cycle. The only season that will be of interest is the expected 12 month season.

### *3.3 Step 2: Autocorrelation Analysis*

The second step is to conduct an autocorrelation analysis of the data using Student Version 5.1 of the software package *MicroTSP* developed by Quantitative MicroSoftware in Irvine, California. *MicroTSP* is an IBM compatible regression and forecasting tool that contains many time series applications. Due to lack of software (at least locally) that can compute sample autocorrelations and sample partial autocorrelations of spatial-temporal data, the sample autocorrelations and sample partial autocorrelations are computed for each of the 22 geographical regions in the temporal dimension. The actual theoretical temporal autocorrelations and temporal partial autocorrelations are unknown and are estimated using the sample temporal autocorrelation function and the sample temporal partial autocorrelations function, respectively. For the remainder of this report, the term autocorrelation is assumed to mean sample temporal autocorrelation and the term partial autocorrelation is assumed to mean sample temporal partial autocorrelation. The autocorrelations are calculated using the following equation:

$$r_k = \frac{\sum_{t=1}^{T-k}[x(t) - \bar{x}][x(t+k) - \bar{x}]}{\sum_{t=1}^{T}[x(t) - \bar{x})^2]} \tag{16}$$

where $r_k$ is the autocorrelation at lag $k$,

$T$ is the number of observations

and $K = \frac{T}{4}$

for $k = 1, 2, \ldots, K$ (16:260).

The partial autocorrelations are calculated using the following equation:

$$r_j = \sum_{i=1}^{k} \phi_{ki} r_{j-i} \tag{17}$$

where $r_j$ is the partial autocorrelation of the $j$th autoregressive process

$\phi_{ki}$ is the $i$th coefficient in an autoregressive process of order $k$,

for $j = 1, 2, \ldots, k$ (16:261).

It is common practice to compute the autocorrelations and partial autocorrelations for the first $\frac{T}{4}$ lags, where $T$ is the total number of observations in the series (16:260). For example, the historical data contains 79 observations for each geographical region and thus, it would be appropriate to calculate autocorrelations and partial autocorrelations for the first 20 lags ($\frac{79}{4} = 19.75$).

The first purpose of autocorrelation analysis is to ensure the data is stationary, which means that the joint distribution of any sequence of observations $x(t+1)$, $x(t+2)$, . . . ,$x(t+n)$ does not depend on $t$. The autocorrelations measure the relationship or how much interdependence exists between neighboring points in a time series.

Stationarity can be detected by examining a plot of the autocorrelations and by conducting ad hoc tests. A plot of the autocorrelations may reveal non-stationarity in the data. There are two ad hoc tests to determine if the data is stationary. The first ad hoc stationarity test is always conducted first. If the times series does not pass the first ad hoc stationarity test, then the second ad hoc stationarity test is conducted. The first ad hoc stationarity test states that it is reasonable to assume the data is stationary if any one of the first three time lag autocorrelation values is not significantly different from zero (12). The second stationarity test states that it is reasonable to assume that the data is stationary if any one of the first two time lag autocorrelation values is significantly different from zero and is followed by a time lag autocorrelation value that is also significant significantly different from zero but opposite in sign (12). If the data fails to pass the stationarity tests, it is reasonable to assume that the data is not stationary. If the data passes any one of the tests, it is reasonable to assume that the data is stationary.

If the data is non-stationary in the mean, the data can be differenced in an attempt to remove the non-stationarity by using the following equation:

$$x(t) = x(t) - x(t-1) \tag{18}$$

It should be noted that first differenced data will lose one degree of freedom. Differencing is a technique that attempts to remove linear trend from the data. Sometimes it is necessary to take multiple differences, which attempts to remove non-linear trend.

The second purpose of autocorrelation analysis is to examine for seasonality. Detection is best seen as a large autocorrelation value that is significantly different from zero. It should be noted that a seasonality of 12 months is expected. Thus, a significant autocorrelation value is expected at the twelfth lag. Another way to detect for seasonality is a repetitive cycle in the autocorrelations.

If the data contains non-stationary seasonality, it is reasonable to assume that the data may be de-seasonalized by taking a difference equal to the number of periods in the season. For example, if the non-stationary season consists of 12 periods, the data can be de-seasonalized using the following equation:

$$x(t) = x(t) - x(t-12) \tag{19}$$

It should be noted that a se[illegible]hat is differenced by 12 periods will lose 12 degrees of freedom.

### 3.4 Step 3: Determination of Target Region, Neighbors, and Spatial Weights

The univariate STARMA model allows for the spatial-temporal modelling of one geographical region termed the"target region." Once the target region is selected, the neighbors are determined exogenously from the data such that first order neighbors influence the target region more than second order neighbors and second order neighbors influence the target region more than the third order neighbors et cetera. In the classical univariate STARMA model, first order neighbors are closer in distance than second order neighbors and second order neighbors are closer in distance than third order neighbors et cetera. Finally, spatial weights are determined to represent the correlation or relationship between the target region and each of its neighbors. In the classical univariate STARMA model, the spatial weights are a function of the distance between the target region and the respective neighbor.

### 3.5 Step 4: Identification

Once the data is stationary, the fourth step of identifying the univariate STARMA model begins. Due to the lack of commercial software packages (at least locally) that develop univariate STARMA models, an ARMA approach will be used to identify the univariate STARMA model (13).

*3.5.1 ARMA Model Building on the Target Region.* The first step is to develop an ARMA model on the target region. The autoregressive parameter $p$ and the moving average parameter $q$ will be calculated using the ARMA forecasting technique of *MicroTSP*. Of course, the stages of identification, estimation, and diagnostic checking of the Box-Jenkins iterative approach must be followed. The forecasting stage need only be conducted after a univariate STARMA model has been successfully through the first three stages. The $p$ and $q$ values from the ARMA model developed on the target region will be used as the actual $p$ and $q$ values in the univariate STARMA model building identification stage. The $p$ value represents the temporal autoregressive order of the univariate STARMA model. Likewise, the $q$ value represents the temporal moving average order of the univariate STARMA model.

The criterion for selected the best ARMA Model for the target region will be that ARMA model that is the best and most parsimonious model that passes diagnostic checking. A best model is defined as a model that adequately describes and represents the time series such that the fraction of the variance explained by the model, $\bar{R}^2$, is maximized and the sum of the squared residuals, SSR, is minimized. A parsimonious model is defined as a model that uses the smallest number of parameters necessary.

*3.5.2 ARMA Model Building on the Spatial Relationships.* After the best and most parsimonious model has been found for the target region, the spatial relationship between the target region and its neighbors is identified.The first step is to energy normalize the weights for each order $l$. In effect, the weights for the $l$th order neighbors of the target region sum to 1.0. The second step is to create a time series consisting of the weighted sum of the observations that are $l$th order neighbors for all $l$. In effect, the $z$ time series of the $l$th order neighbors are converted into a single time series using the following relationship:

$$Y_l(t) = \sum_{j=1}^{z} W_{jl} x_j(t) \ for \ l = 1, 2, ..., L \tag{20}$$

where $Y_l(t)$ is the weighted sum of the $l$th order neighbors at time $t$,

$z$ is the number of $l$th order neighbors to the target region

$W_{ijk}$ is the weight given to the spatial relationship between target region and neighbor $j$ of order $l$

$x_j(t)$ is the observation of the $j$th neighbor of order $l$ at time $t$,

$L$ is the largest neighbor order, and

the weights sum to unity for each order $l$ (13:22).

The weights are defined for an order $l$ when the target region and region $j$ are neighbors (13:22).

The next step creates a single time series containing the times series of the target region and the $l$ time series of weighted neighbors. The time series $Y_0(t)$ is the target region time series. Table 1 shows how the single time series is formed where $n$ is the total number of observations.

Table 1. Creation of Combined Time Series

| COMBINED TIME SERIES |
|---|
| $Y_0(t=1)$ |
| $Y_1(t=1)$ |
| $Y_2(t=1)$ |
| . |
| $Y_y(t=1)$ |
| $Y_0(t=2)$ |
| $Y_1(t=2)$ |
| $Y_2(t=2)$ |
| . |
| $Y_y(t=2)$ |
| . |
| . |
| . |
| . |
| $Y_0(t=n)$ |
| $Y_1(t=n)$ |
| $Y_2(t=n)$ |
| . |
| $Y_y(t=n)$ |

The Box-Jenkins Autoregressive Moving Averages (ARMA) forecast program in *MicroTSP* will calculate the $k$th spatial autoregressive order $\lambda_k$ and the $k$th spatial MA moving averages order $m_k$. This is accomplished by developing an ARMA model on the combination series where the value of $p$ corresponds to the value of the identified $m_k$ and the value of $q$ corresponds to the value of the identified $\lambda_k$.

If the best and most parsimonious model found in the ARMA model building of the target region was a pure AR or seasonal autoregressive (SAR) process, it is not necessary to calculate the spatial moving average $m_k$ value because $m_k$ is assumed to be equal to zero. If the best and most parsimonious model found in the ARMA model building of the target region was a pure MA or seasonal moving averages (SMA) process, it is not necessary to calculate the spatial autoregressive $\lambda_k$ value because $\lambda_k$ is assumed to be equal to zero. If the best and most parsimonious ARMA model found for the target region was an ARMA or seasonal autoregressive moving averages (SARMA) model, both the $m_k$ and the $\lambda_k$ values must be calculated. However, it is still quite possible that $m_k$ or $\lambda_k$ will equal zero though an ARMA or SARMA model was specified for the target region.

Of course, the ARMA model building process to determine the spatial relationships between the target region and its neighbors must follow the first three stages of the Box-Jenkins iterative approach. The $p$ and $q$ values of the best and most parsimonious model will be used as the $m_k$ and $\lambda_k$ values, respectively, in the univariate STARMA model building identification stage. The $p$ value represents the spatial autoregressive order $\lambda_p$ of the univariate STARMA model. Likewise, the $q$ value represents the spatial moving average order $m_q$ of the univariate STARMA model.

The criterion for selecting the best ARMA Model for the spatial relationships will be the same as that of the ARMA model on the target region. If either the $m_k$ value or the $\lambda_k$ value is non-zero, there is a spatial relationship between the target region and its neighbors and it is appropriate to continue modelling the data as a univariate STARMA model. If both the $m_k$ value and the $\lambda_k$ value are zero, there is no spatial relationship and thus, a univariate STARMA model is not be appropriate for the data and the specified weights.

### *3.6 Step 5: Estimation.*

Once the values for $p$, $q$, $\lambda_k$ and $m_k$ have been identified through the ARMA modelling, the estimation of the univariate STARMA model parameters is conducted. The Box-Jenkins program of *MicroTSP* will estimate all univariate STARMA parameters using the combination series.

### 3.7 Step 6: Diagnostic Checking.

Diagnostic checking of the univariate STARMA model insures the coefficients of the parameters are significantly different from zero. The diagnostic checking also ensures that $\epsilon(t)$, the error terms, are white noise. *MicroTSP* calculates the $t$-statistic and the two tailed significance of each of the coefficients. Assuming a 90% confidence level, a coefficient is significantly different from zero if its corresponding two tailed significance is less than or equal to 0.10.

To ensure that $\epsilon(t)$, the error terms, are white noise, a plot of $\epsilon(t)$ will be generated. The autocorrelations and partial autocorrelations of $\epsilon(t)$, along with the $Q$-statistic of the sample autocorrelations, will be calculated using *MicroTSP*. If the autocorrelations pass the stationarity test, then $\epsilon(t)$ is white noise.

### 3.8 Step 7: Forecasting.

The first step in the forecasting stage is to select the best and most parsimonious model. The best and most parsimonious model is that model which uses the smallest number of parameters necessary to adequately describe and represent the $N$ time series such that the $\bar{R}^2$ is maximized and the SSR is minimized. The best and most parsimonious model is desired to keep the model as simple as possible. One way to keep the model simple is to include as few parameters as is necessary to adequately represent the data. Once the best and most parsimonious model is selected, the model can be used to predict the $N$ time series.

# *IV. Results and Analysis*

## *4.1 Introduction*

The development of a STARMA model is an iterative process. Several steps were repeated. This section summarizes each step as it occurred in the iterative process.

## *4.2 Step 1: Data Analysis of the Historical Data*

*4.2.1 Three Dimensional Plots of the Historical Data.* Three dimensional plots using several perspectives of the historical data with time on the $x$-axis, geographical region on the $y$-axis and $X_{2j1}$ on the $z$-axis were examined. The historical data plots can be found in Figure 1, Figure 2, and Figure 3. The $x$-axis is set up such that $x = 1$ corresponds to January 1985, $x = 2$ corresponds to February 1985, ..., and $x = 79$ corresponds to July 1991. The $y$-axis is set up such that $y = 1$ corresponds to geographical region 1, $y = 2$ corresponds to geographical regions 2, ..., and $y = 22$ corresponds to geographical region 22. The values for the historical data can be found in Appendix A.

The different perspectives allow for viewing of the three-dimensional plots from several view points. The perspective controls the way the three-dimensional coordinates of the plot are mapped into the two-dimensions by setting the rotation angles (in degrees) (26:29-30). Six different perspectives are used in this paper. Perspectives 1, 2, and 3 are used on three-dimensional plots of data such as the historical data base. Perspectives A, B, and C are used on three-dimensional plots of autocorrelations and partial autocorrelations. Perspective 1 is aligned such that the $x$-axis is rotated 10 degrees and the $y$-axis is rotated 30 degrees. Perspective 2 is aligned such that the $x$-axis is rotated 10 degrees and the $y$-axis is rotated 60 degrees. Perspective 3 is aligned such that the $x$-axis is rotated 10 degrees and the $y$-axis is rotated 80 degrees. Perspective A is aligned such that the $x$-axis is rotated 30 degrees and the $y$-axis is rotated 30 degrees. Perspective B is aligned such that the $x$-axis is rotated 30 degrees and the $y$-axis is rotated 60 degrees. Perspective C is aligned such that the $x$-axis is rotated 30 degrees and the $y$-axis is rotated 80 degrees.

It is very obvious from Figures 1, 2, and 3 of the historical data using several perspectives that there are numerous observations that have a $X_{2j1}$ value of 0.0. Because the observations are relative frequencies, the observation values are limited to range between 0.0 and 1.0. Geographical regions 7, 9, and 11 appear to have the most nonzero $X_{2j1}$ values. According to the experts on the historical data base, the odd numbered regions are more likely to have non-zero $X_{2j1}$ values than the even numbered regions (2). Few nonzero observations occur in geographical regions 1, 2, 3, 4, 5, 6, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, and 22. The year 1986 had few nonzero $X_{2j1}$ values.

*4.2.2 Two Dimensional Plots of the Average Relative Frequency for Each Year.* Two dimensional plots of the average relative frequency of the historical data base for each year were examined. The two-dimensional plots for each year can be found in Figures 4 through 10. The $x$-axis is set up such that $x = 1$ corresponds to Region 1, $x = 2$ corresponds to Region 2, ..., and $x = 22$ corresponds to Region 22. The values on the $y$-axis are the average relative historical frequencies for the given year for the corresponding region. The values for the average relative frequency for each year can be found in Appendix B.

It should be noted that the data collected for 1991 in Figure 10 included data only up through July. The purpose behind examining these plots is to see if there is a trend in the data. The average relative frequencies for each year were different but there did not appear to be any pattern or trend to the changes over the years. One thing of interest is that the geographical region that exhibited the largest average relative frequency for each year changed over the years. Table 2 lists the geographical region that exhibited the largest average relative frequency for the respective year.

NOTE: The year 1991 only includes information through July.

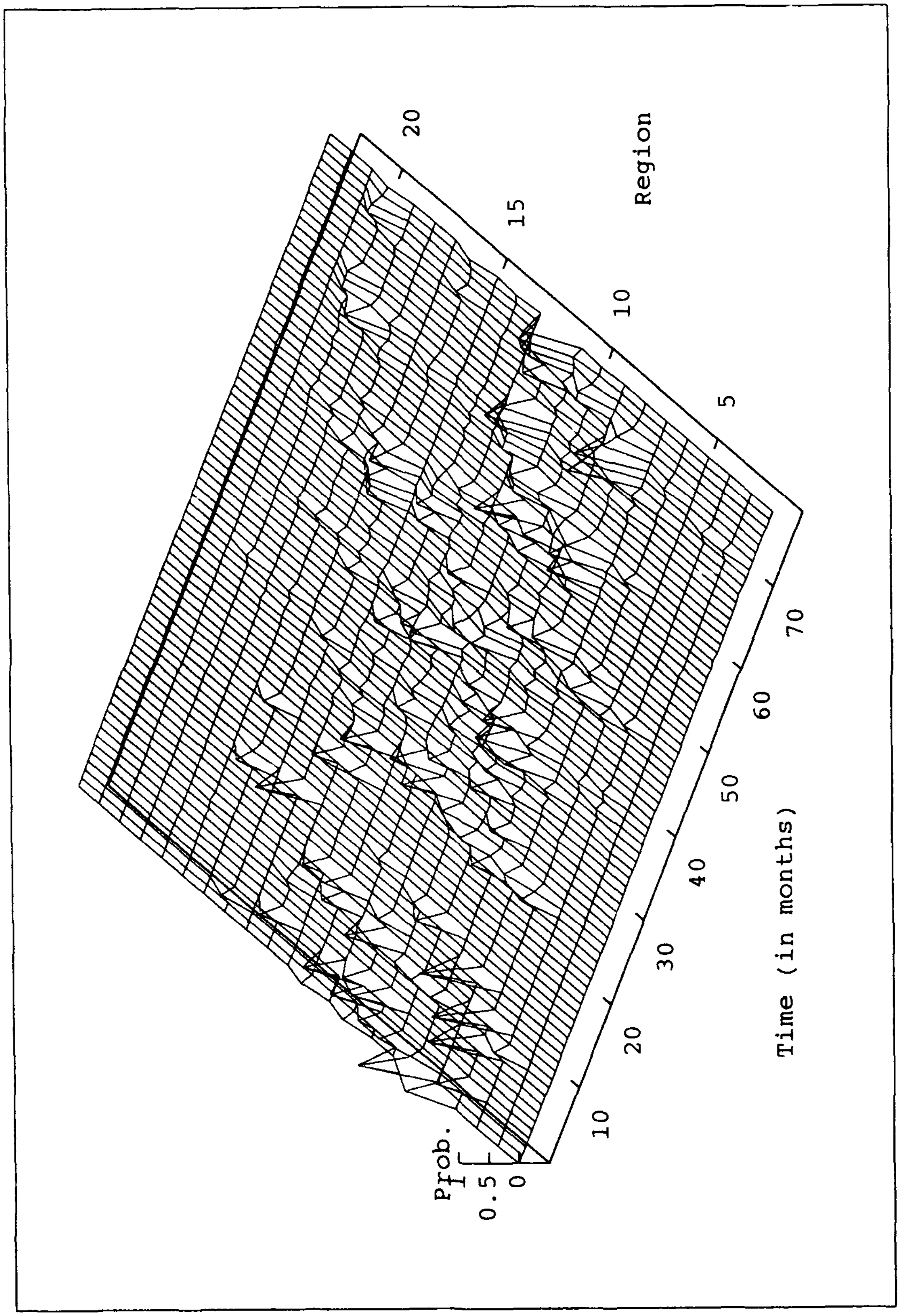


Figure 1. Historical Data $X_{2j1}$ from Perspective 1

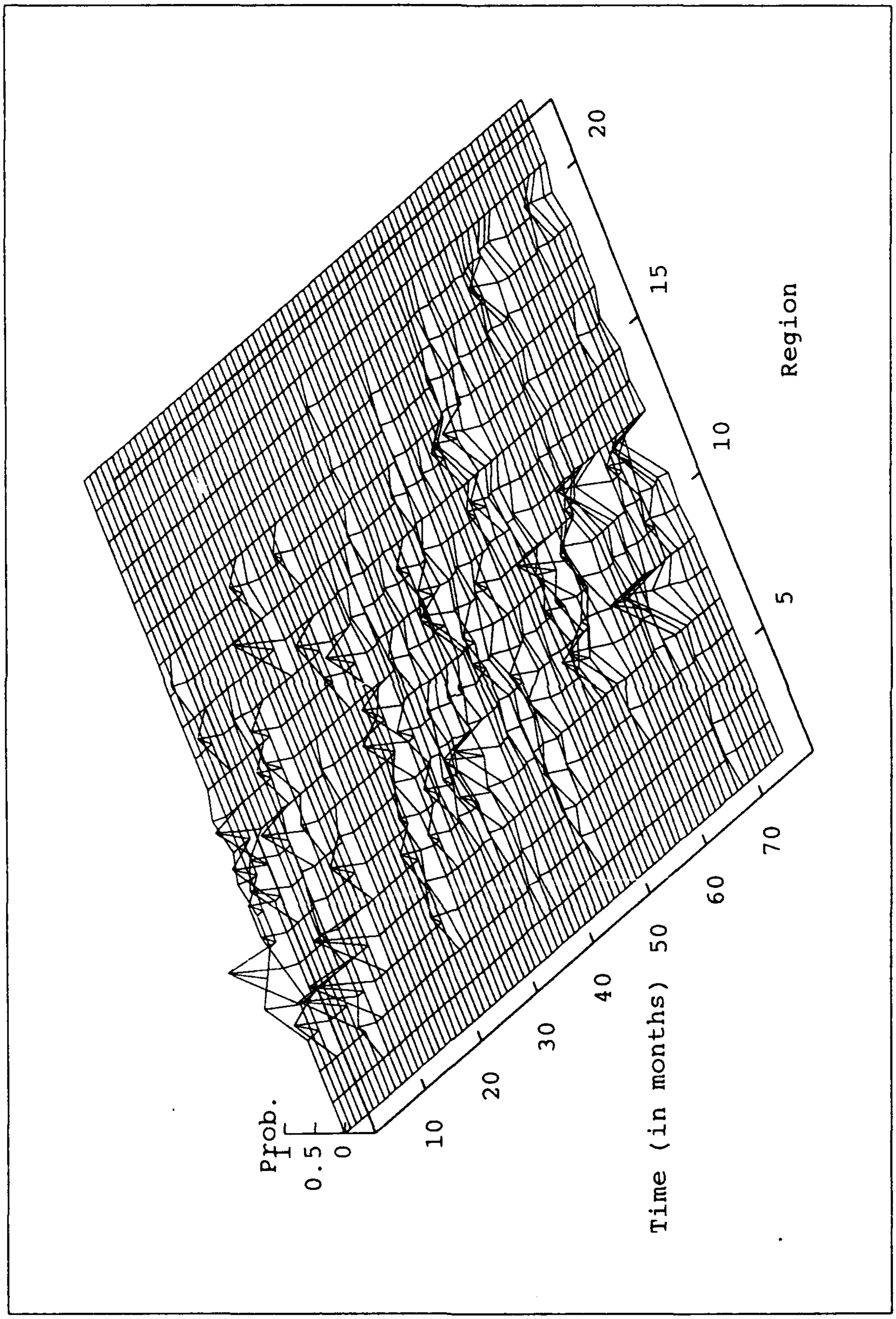


Figure 2. Historical Data $X_{2j1}$ from Perspective 2

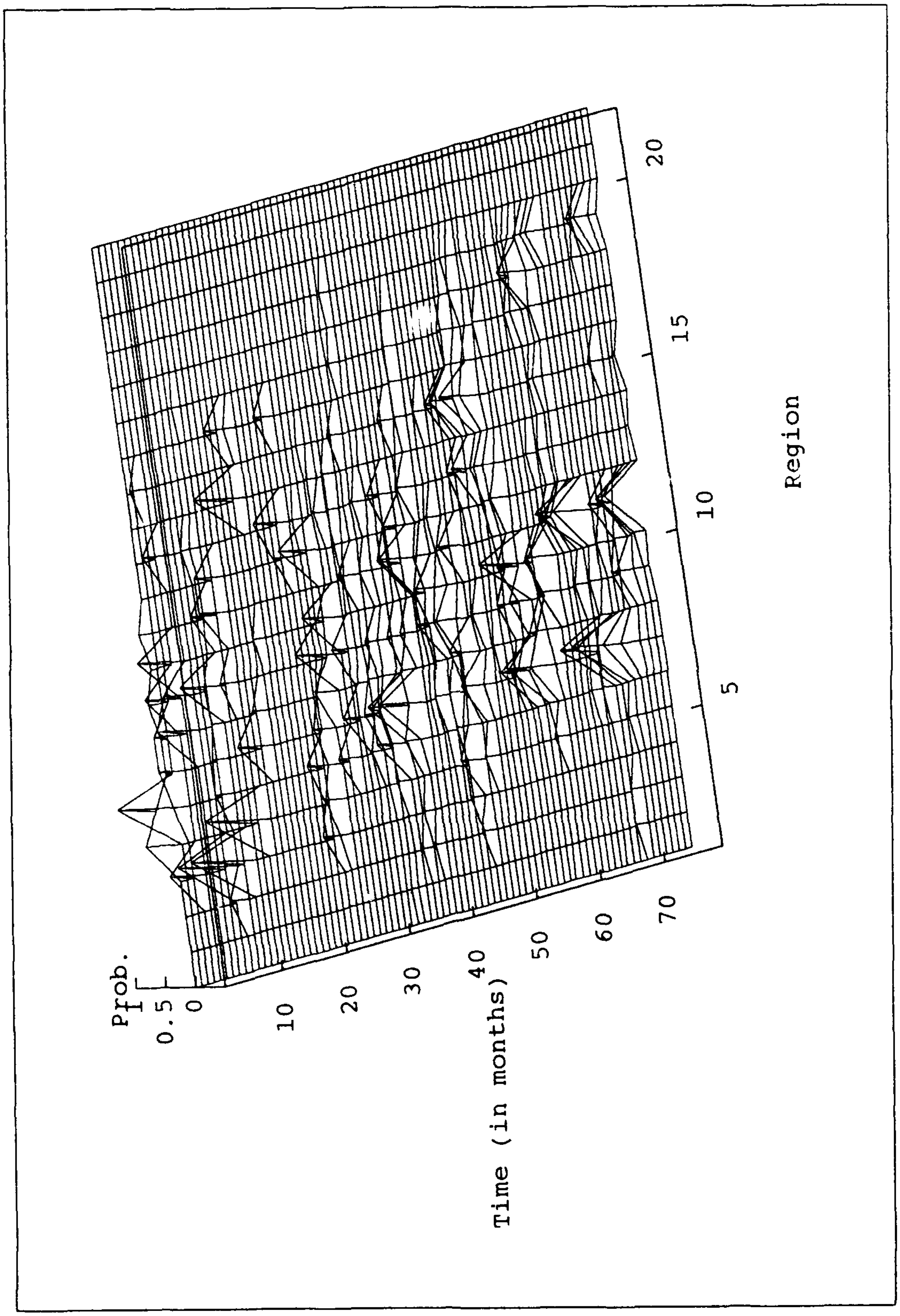


Figure 3. Historical Data $X_{2j1}$ from Perspective 3

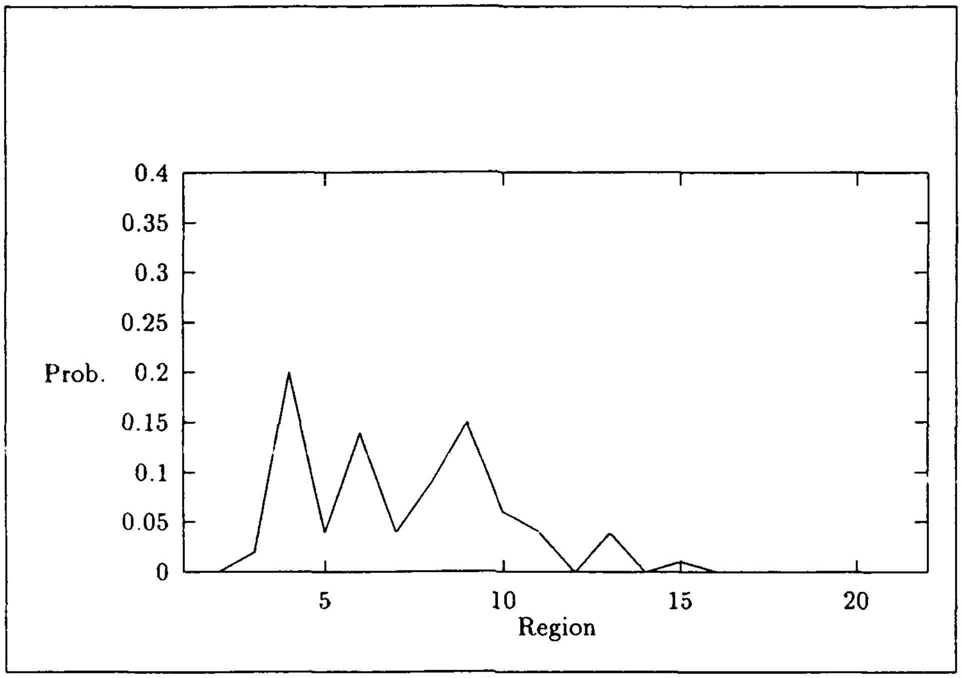


Figure 4. Average Relative Frequency in 1985

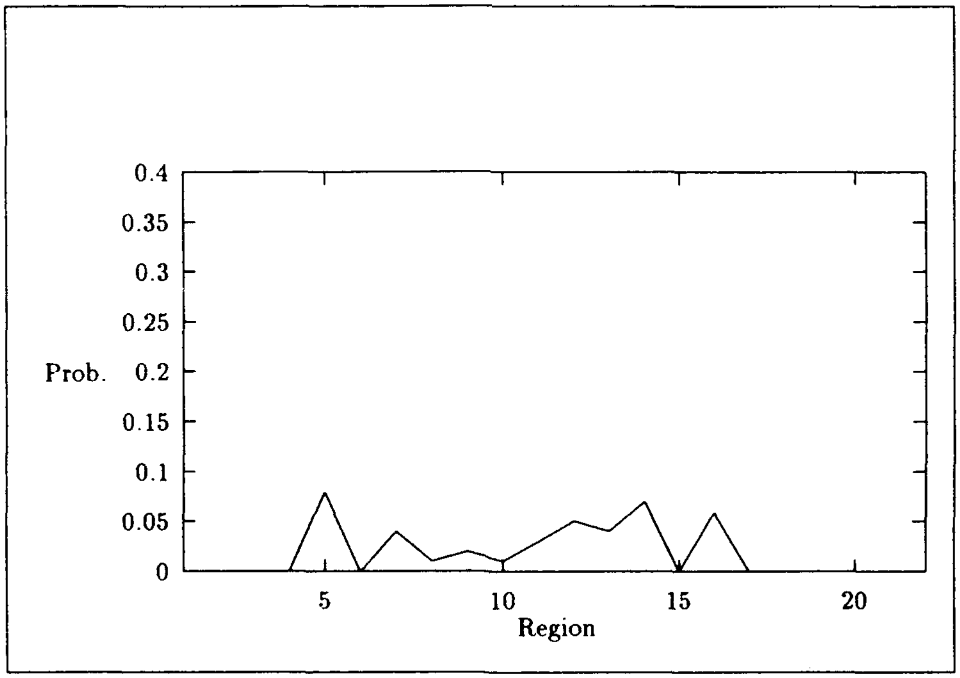


Figure 5. Average Relative Frequency in 1986

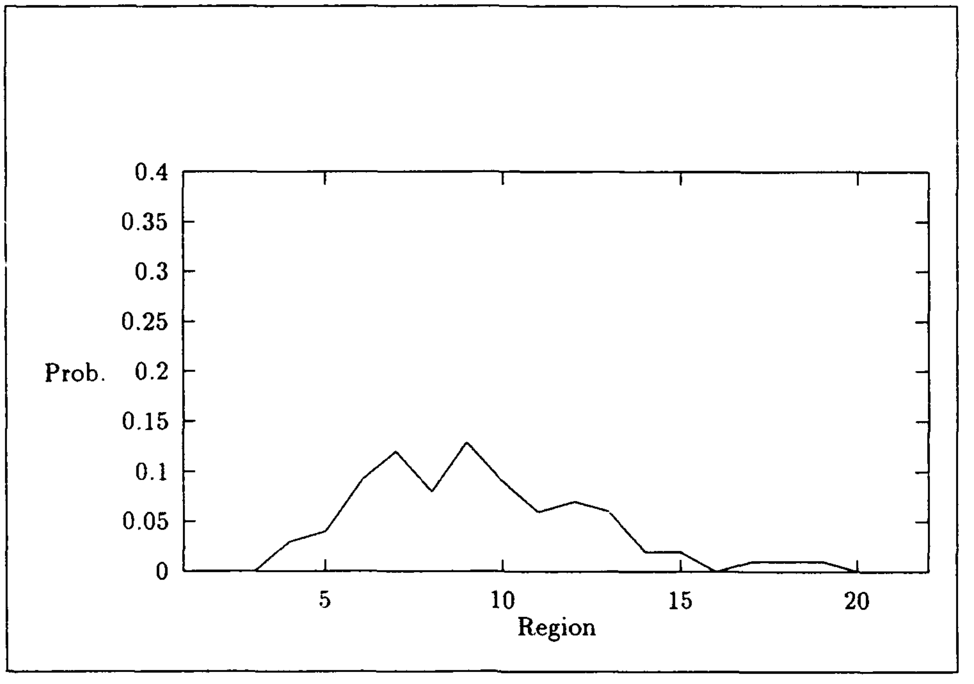


Figure 6. Average Relative Frequency in 1987

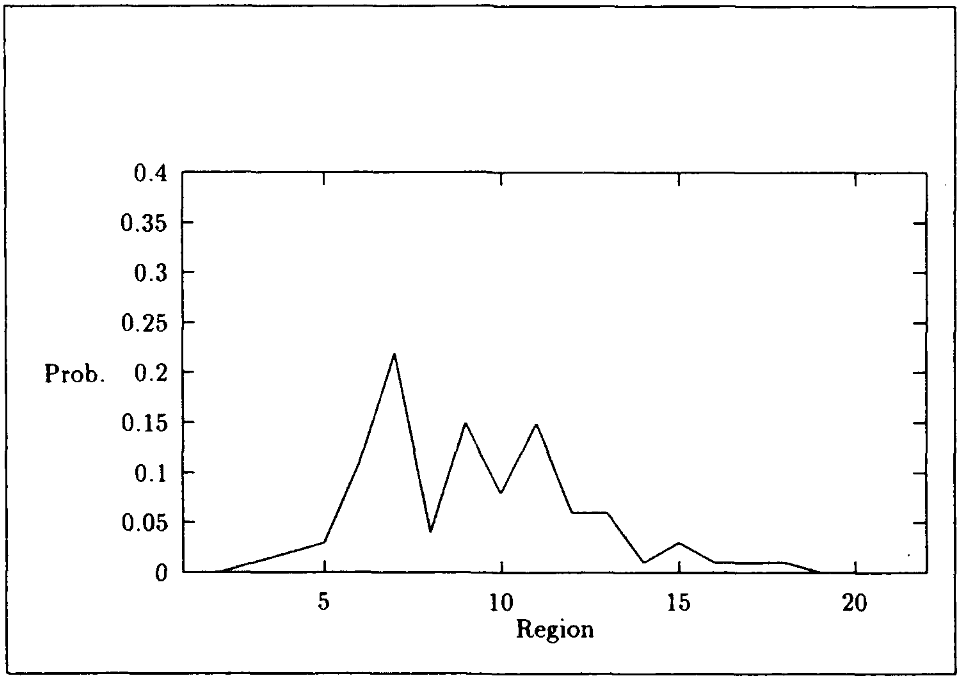


Figure 7. Average Relative Frequency in 1988

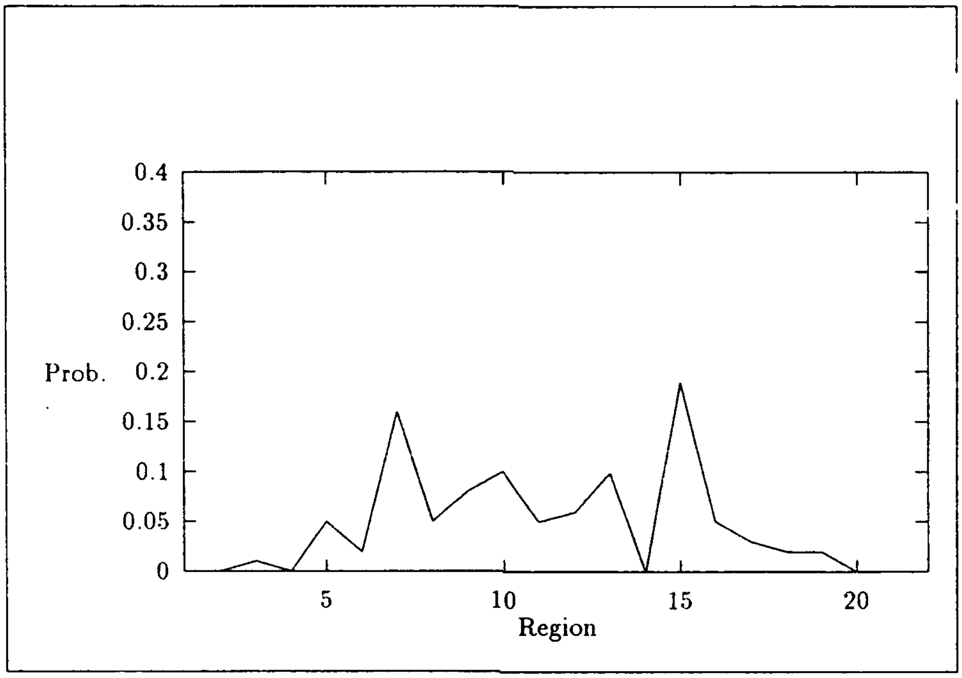


Figure 8. Average Relative Frequency in 1989

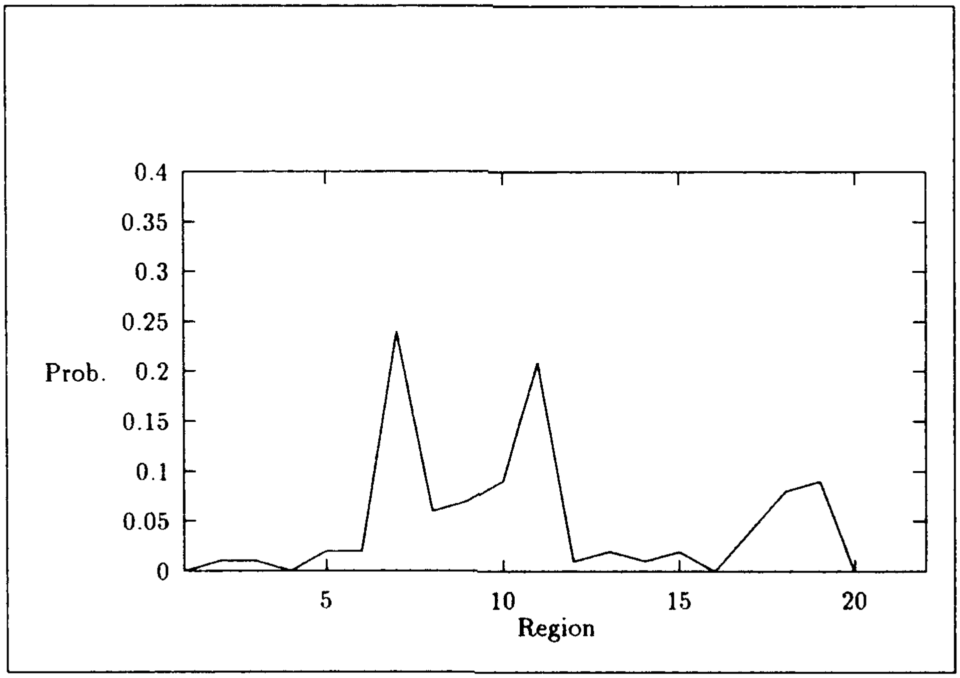


Figure 9. Average Relative Frequency in 1990

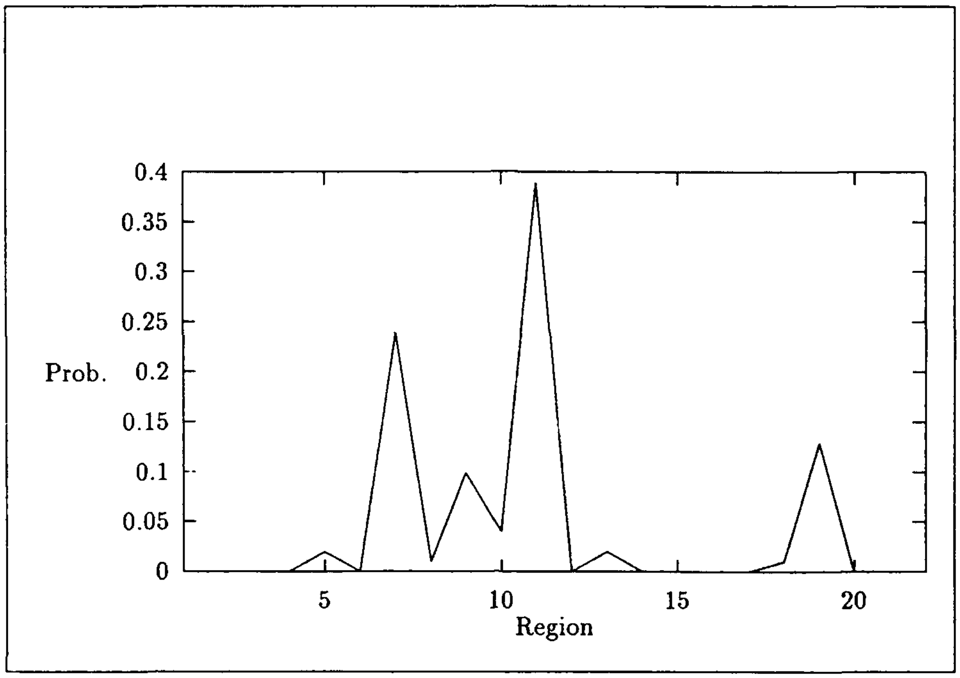


Figure 10. Average Relative Frequency in 1991

Table 2. Region Exhibiting the Largest Average Relative Frequency

| Year | Region |
|---|---|
| 1985 | 9 |
| 1986 | 16 |
| 1987 | 7, 8 (tie) |
| 1988 | 9 |
| 1989 | 8 |
| 1990 | 11 |
| 1991 | 11 |

*4.2.3 Two Dimensional Plots of the Average Relative Probabilities for Each Season.* The next set of plots examined were two-dimensional plots of the average relative historical frequencies for each of the four seasons: winter, spring , summer, and fall. The two-dimensional plots for each season can be found in Figures 11 through 14. The values for the average relative historical frequencies foı each of the seasons can be found in Appendix C.

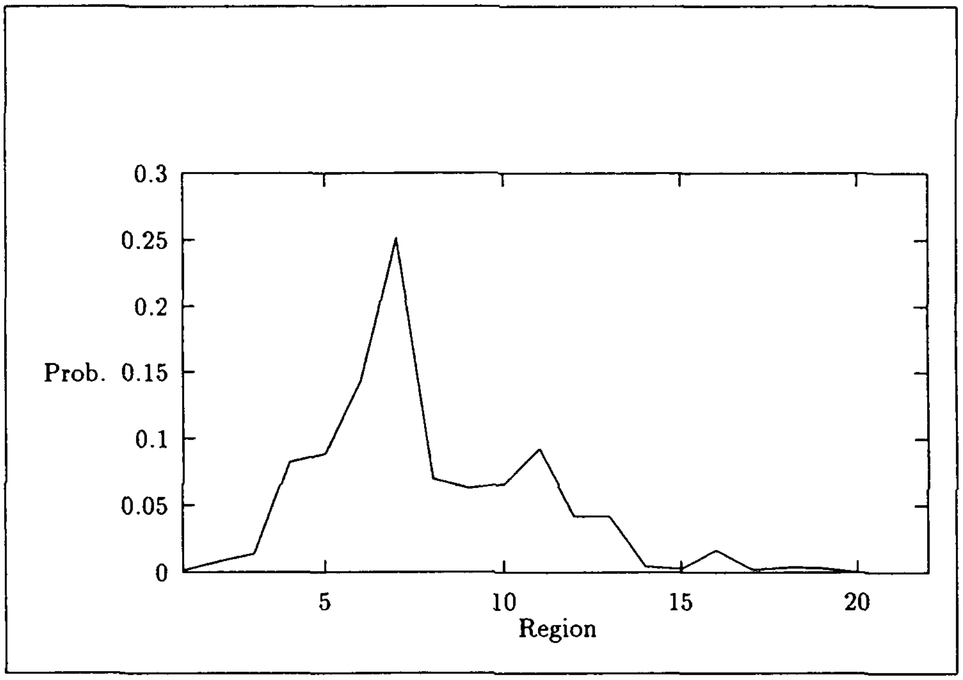


Figure 11. Average Relative Frequency in Winter

There appears to be a change in the average relative frequencies for each season. The average relative frequencies during the winter in Figure 11 appear to favor geographical regions 2 through 13. The three regions that have the largest relative frequency during the winter season are regions 5, 6, and 7. The spring average relative frequencies in Figure 12 appear to be more evenly spread out over the geographical regions than the winter average relative frequencies. The three regions that have the largest average relative frequencies in the spring are regions 5, 7, and 15. The summer average relative frequencies in Figure 13 appears to shift over to the right emphasizing geographical regions 8 through 19. The three largest average relative frequencies in the summer occur at regions

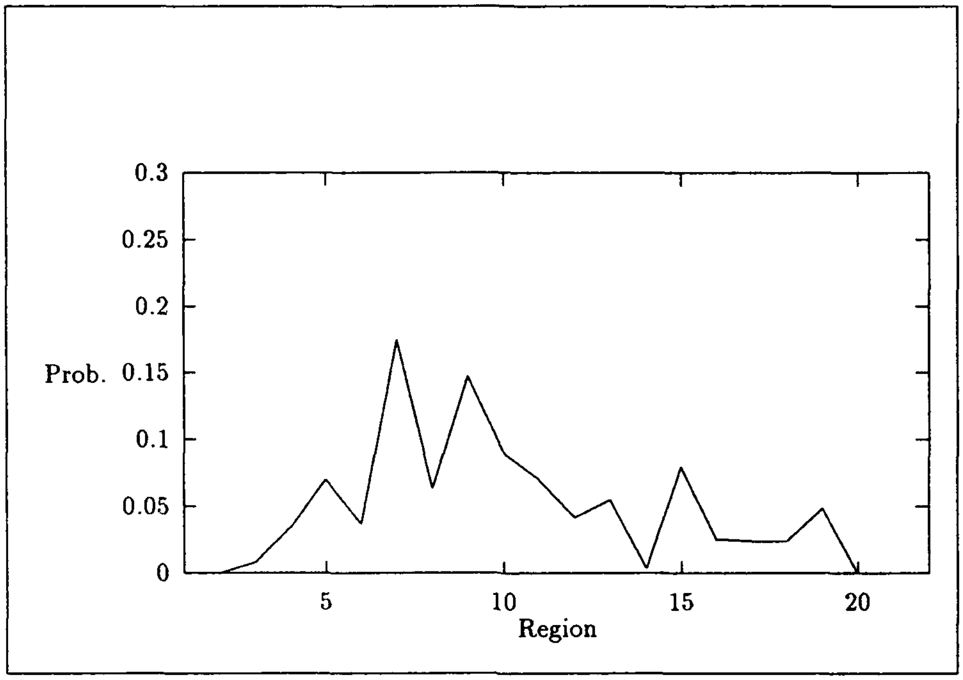


Figure 12. Average Relative Frequency in Spring

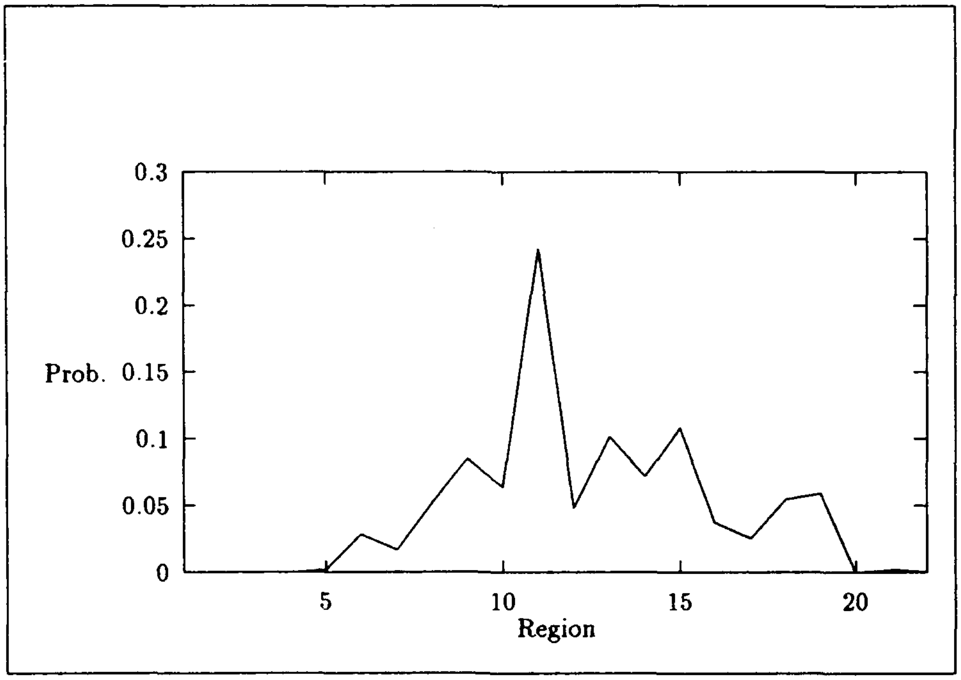


Figure 13. Average Relative Frequency in Summer

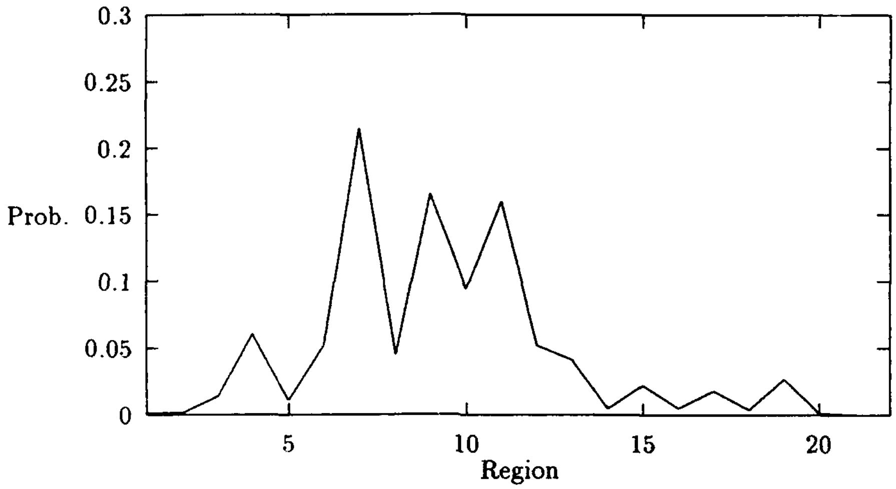


Figure 14. Average Relative Frequency in Fall

11, 13, and 15. The fall average relative frequencies in Figure 14 appear to shift to the left from the summer average relative probabilities but is not shifted over to the left as much as the winter average relative frequencies. The three largest average relative frequencies in the fall correspond to regions 7, 9, and 11. From the plots of the average relative frequencies for each season, it can be concluded that there is seasonality in the historical data base.

### *4.3 Transformations on the Historical and Analytical Data Bases*

There are few non-zero relative frequencies in the historical data base. Because the observations are frequencies, they range between 0.0 and 1.0. The numerous zero $p_{2j1}$ values combined with the limitation on the range of the relative frequencies will make it very difficult to model the historical data base. The analytical estimates can be used as a simple filter on the historical data base. The values for the analytical predictions can be found in Appendix D. This filtering is accomplished simply by subtracting the analytical estimates from the corresponding relative historical frequencies. This difference between the historical relative frequencies and the analytical model predictions can be interpreted as the difference between what was observed to occur what was predicted to occur.

Before calculating the difference, the analytical probabilities were energy normalized, as opposed to statistically normalized, so the probabilities became relative probabilities (statistical normalization changes the highest observation value to 1.0 and the lowest observation value to 0.0 and adjusts for the other observations in relationship to the highest observation value and the lowest observation value). Energy normalization transformed the analytical probabilities such that the sum of the probabilities over the 22 geographical regions summed to one for every time period using the following equation:

$$\breve{p}_{2j1,trans} = \frac{\breve{p}_{2j1}}{\sum_{r=1}^{22} \breve{p}_{2r1}} \qquad (21)$$

for all $j$, $j = 1, 2, \ldots, 22$ over all 79 months

where $r$ indexes the geographical regions.

This was done because the historical frequencies are relative frequencies that sum to one for every time period over the 22 geographical regions. This transformation, in effect, changed the analytical probabilities to relative analytical probabilities. The values of the normalized analytical predictions can be found in Appendix E. The differences were calculated by the following equation:

$$NHA_{2j1} = p_{2j1} - \breve{p}_{2j1,trans} \qquad (22)$$

where NHA is the *N*ormalized *H*istorical frequency minus the *A*nalytical probability for event type 2 at time block 1 for all $j$, $j = 1, 2, \ldots, 22$ for all 79 months .

The difference can be interpreted as information of what was observed that was not adequately represented or explained by the normalized analytical model. For a given month, the difference is positive in sign when the observed relative frequency for $j$ is greater than the predicted analytical relative probability for $j$. This occurs when the analytical model underestimated the relative frequency of occurrence. For a given month, the difference is negative in sign when the observed relative frequency is less than the normalized analytical estimate for $j$. This occurs when the analytical model overestimated the relative frequency of occurrence. Values of $NHA_{2,j,1}$ can be found in APPENDIX F.

### *4.4 Step 1: Data Analysis on the Transformed Data Set $NHA_{1j2}$*

Three-dimensional plots of $NHA_{2j1}$ from several perspectives were examined. The three-dimensional plots can be found in Figure 15, Figure 16, and Figure 17. The $x$-axis is set up such that $x = 1$ corresponds to January 1985, $x = 2$ corresponds to February 1985, ..., and $x = 79$ corresponds to July 1991. The $y$-axis is set up such that $y = 1$ corresponds to region 1, $y = 2$ corresponds to region 2, ..., and $y = 22$ corresponds to region 22.

The $NHA_{2j1}$ values do not appear to be white noise because a 12 month season appears to be present. For example, region 7 has relatively high $NHA_{2j1}$ values during the winter months (December, January, and February) and relatively low $NHA_{2j1}$ values during the summer months (June, July, and August). The winter months that exhibit the relatively high $NHA_{2j1}$ values for region 7 can be seen when $x = 36$, $x = 48$, $x = 60$, and $x = 72$. The summer months that exhibit the relatively low $NHA_{2j1}$ values for region 7 can be seen when $x = 42$, $x = 43$, $x = 44$, $x = 54$, $x = 55$, $x = 56$, $x = 66$, $x = 67$, $x = 68$, $x = 78$, and $x = 79$.

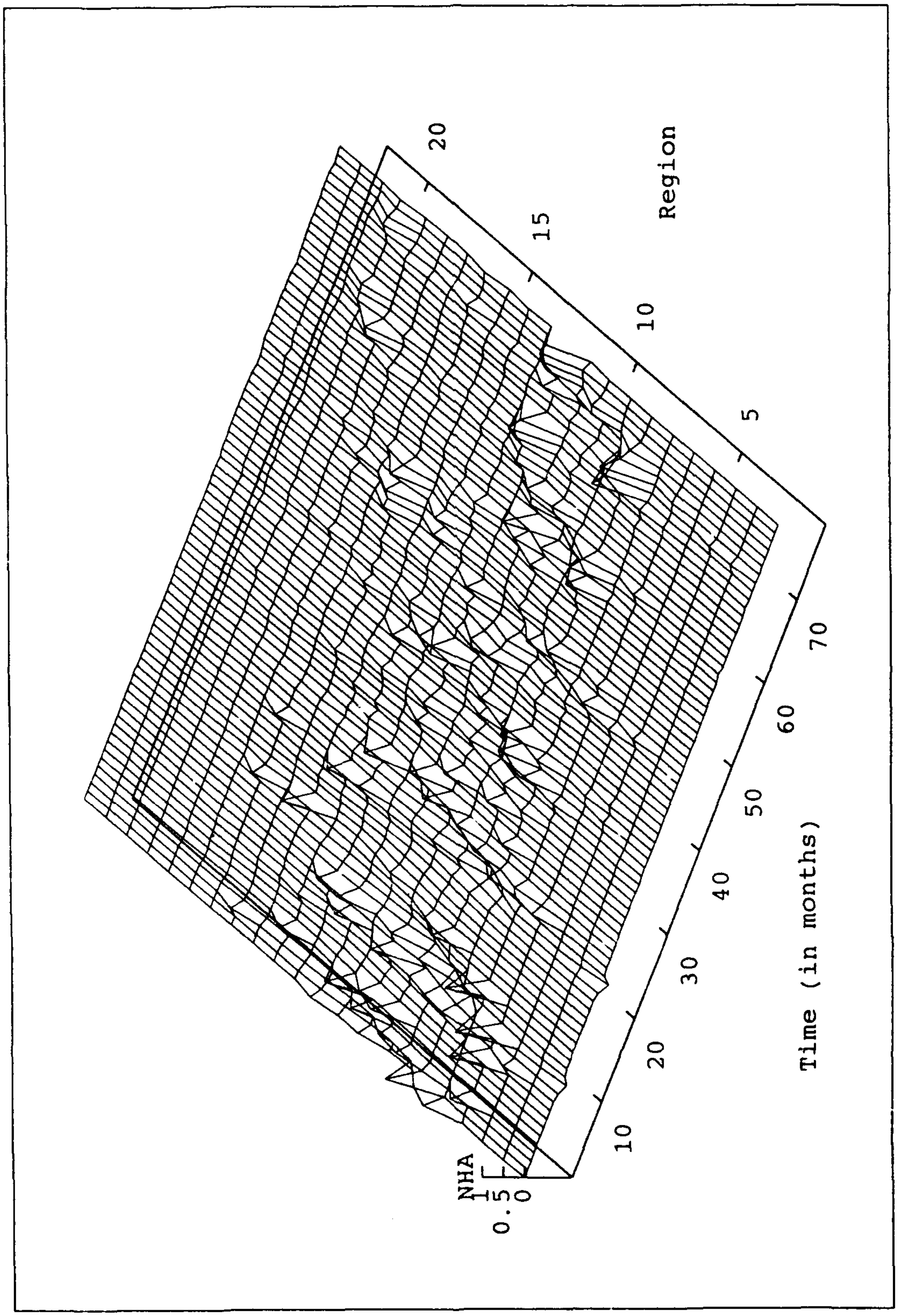


Figure 15. Normalized Historical - Analytical Data $NHA_{2j1}$ from Perspective 1

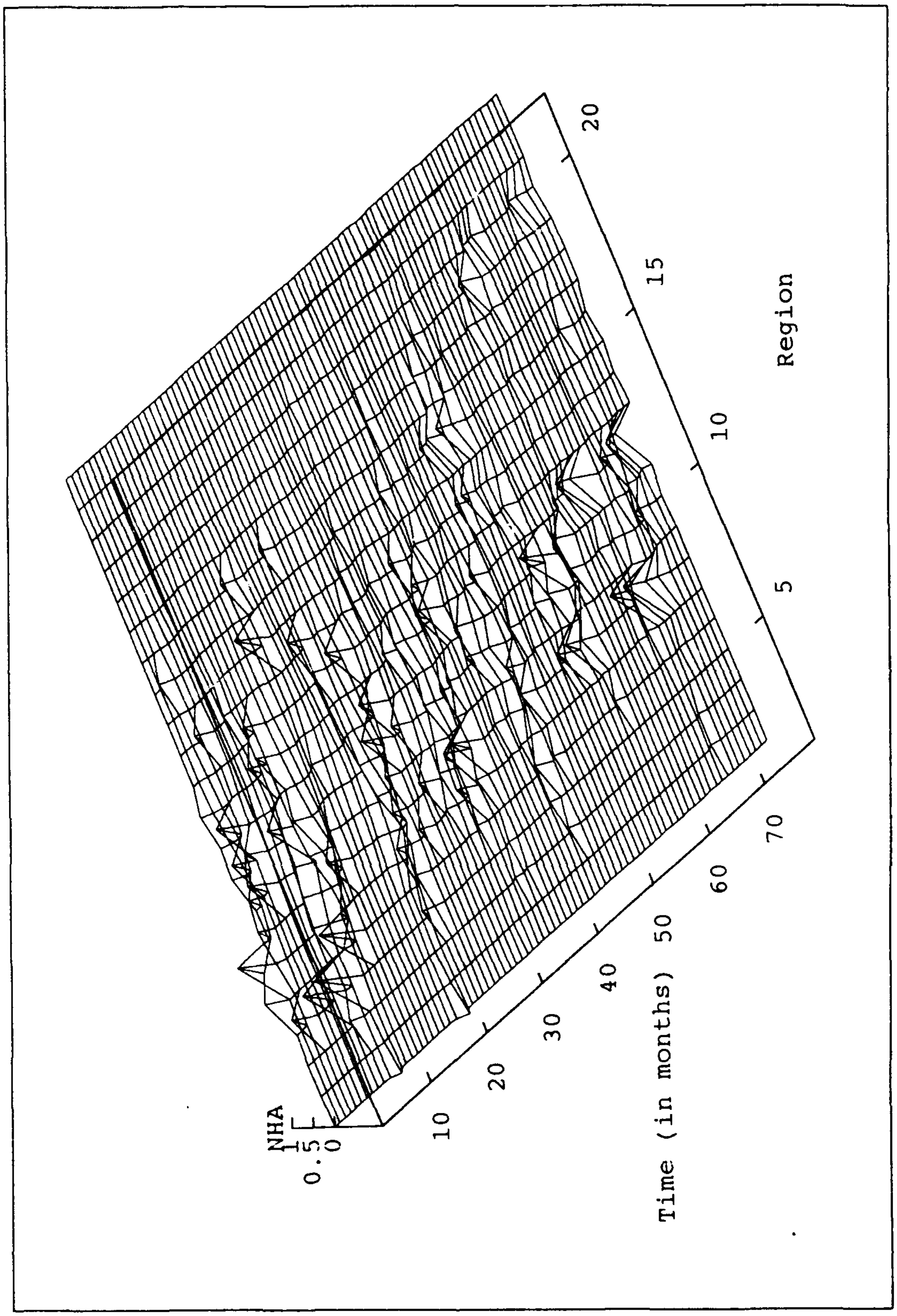


Figure 16. Normalized Historical - Analytical Data $NHA_{2j1}$ from Perspective 2

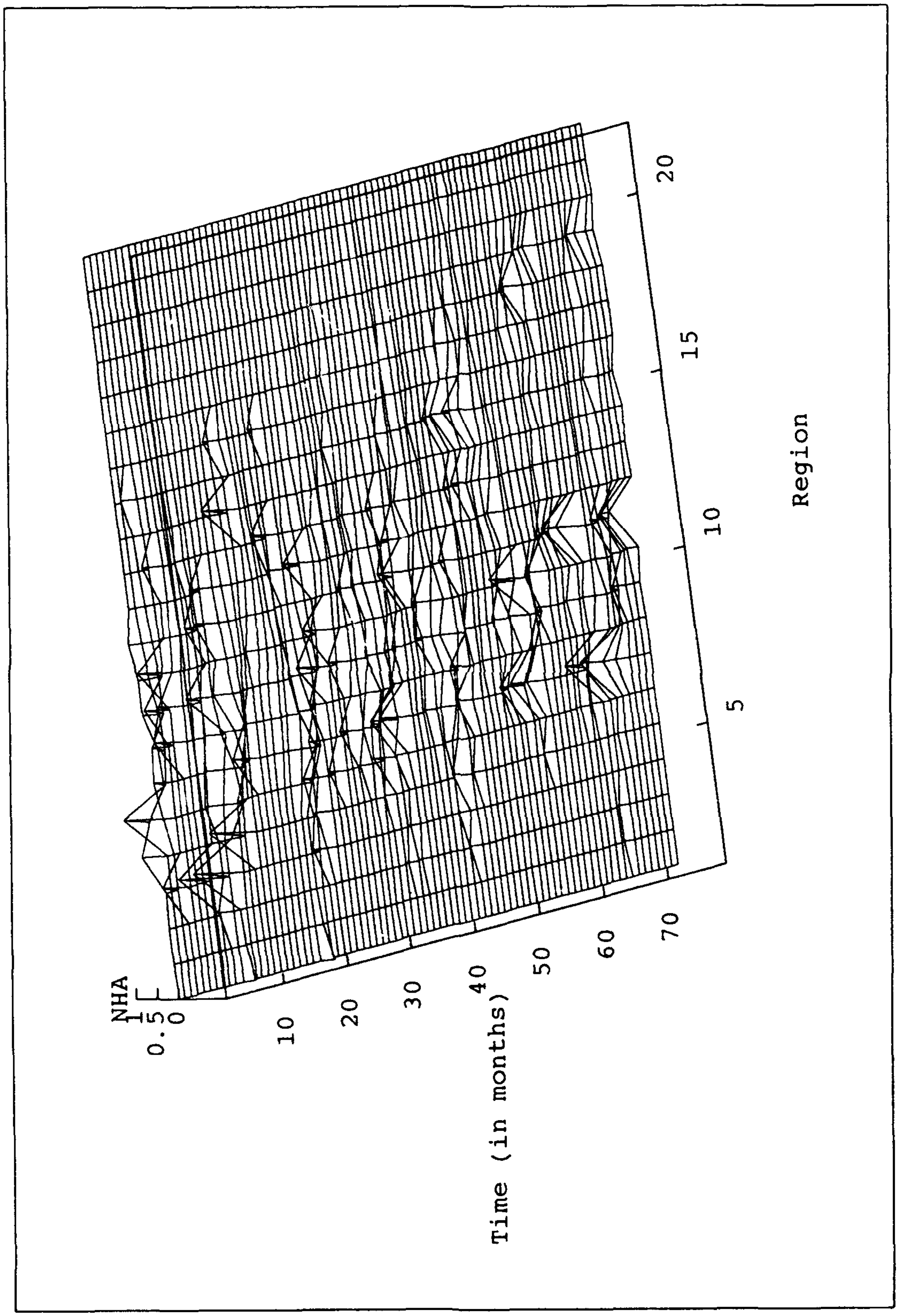


Figure 17. Normalized Historical - Analytical Data $NHA_{2j1}$ from Perspective 3

Several regions become "prominent". Regions 7 and 11 appear to very "mountainous" having many relatively large values of $NHA_{2j1}$. This implies that the analytical model does not adequately predict regions 7 and 11. Because the majority of the values of $NHA_{2j1}$ are positive, this implies that the analytical model's relative estimates greatly underestimate the $p_{2j1}$ values for regions 7 and 11.

### *4.5 Step 2: Autocorrelation Analysis on $NHA_{2j1}$*

An autocorrelation analysis was conducted on the values of $NHA_{2j1}$. Figure 18, Figure 19,and Figure 20 are three-dimensional plots of the autocorrelations of $NHA_{2j1}$ from several perspectives. The $x$-axis is set up such that $x = 1$ corresponds to lag 1, $x = 2$ corresponds to lag 2, ..., and $x = 20$ corresponds to lag 20. The $y$-axis is set up such that $y$ =1 corresponds to region 1, $y = 2$ corresponds to region 2, ..., and $y = 22$ corresponds to region 22. The values of the autocorrelations for $NHAS_{2,j,1}$ can found in APPENDIX G.

Table 3 summarizes the number of regions that had an autocorrelation value significantly different from zero at the $s$ lag.

Six of the regions had an autocorrelation value at the 11th and the 12th lags that are significantly different from zero. This implies that a 12 month season may be present in the data. Experts familiar with the historical data base suggest that a cycle of 11 years in present in the data. Thus, it is assumed that the 12 month season is non-stationary as a result of the 11 year cycle.

### *4.6 Transformation on the $NHA_{2j1}$ Values to Remove Seasonality*

Because the 12 month seasonality is assumed to be non-stationary, differencing can be applied to remove the seasonality. To remove the 12 month season, the values $NHA_{2j1}$ were differenced using a 12 month lag. This was accomplished by the following equation:

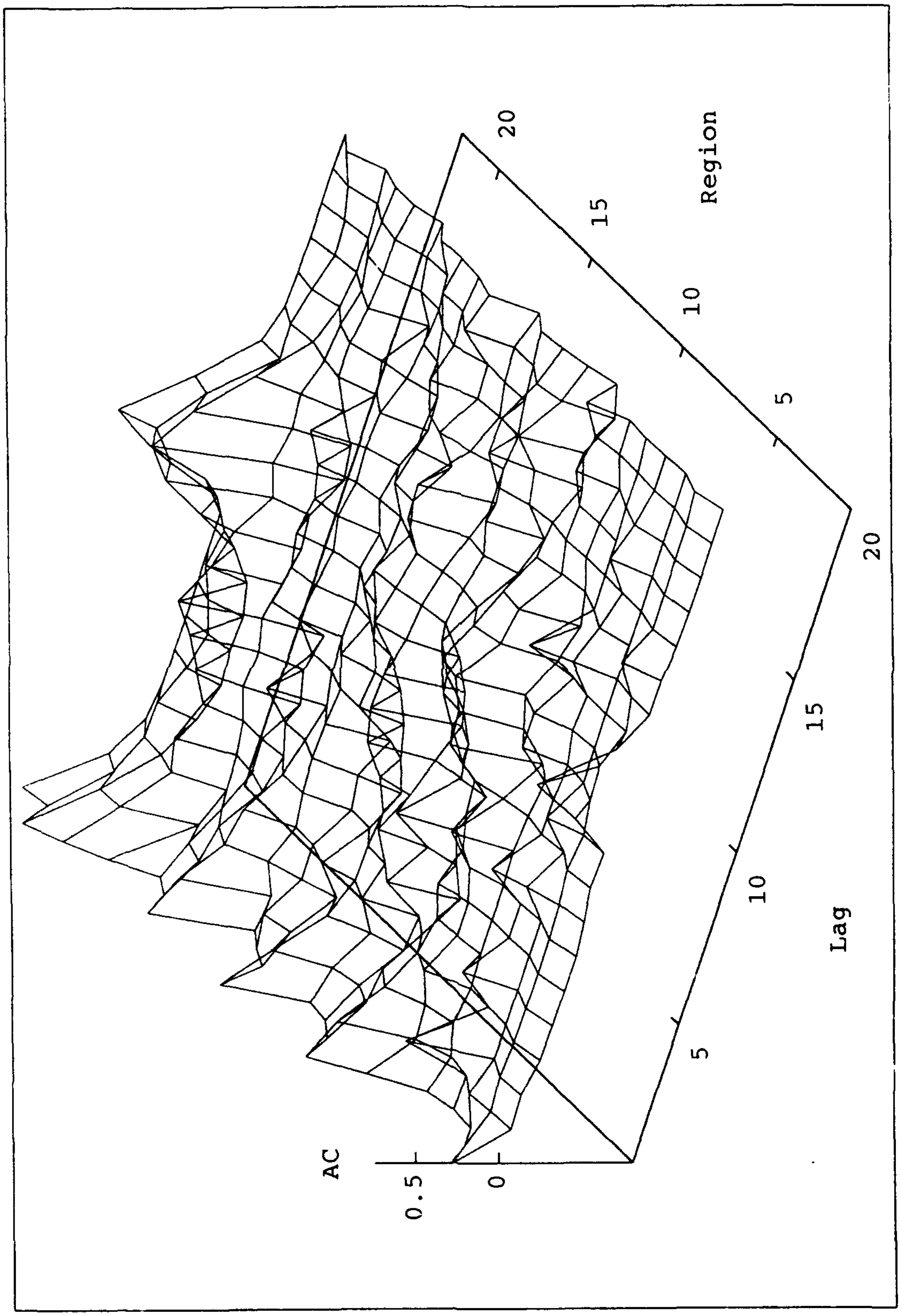


Figure 18. Autocorrelations of $NHA_{2j1}$ from Perspective A

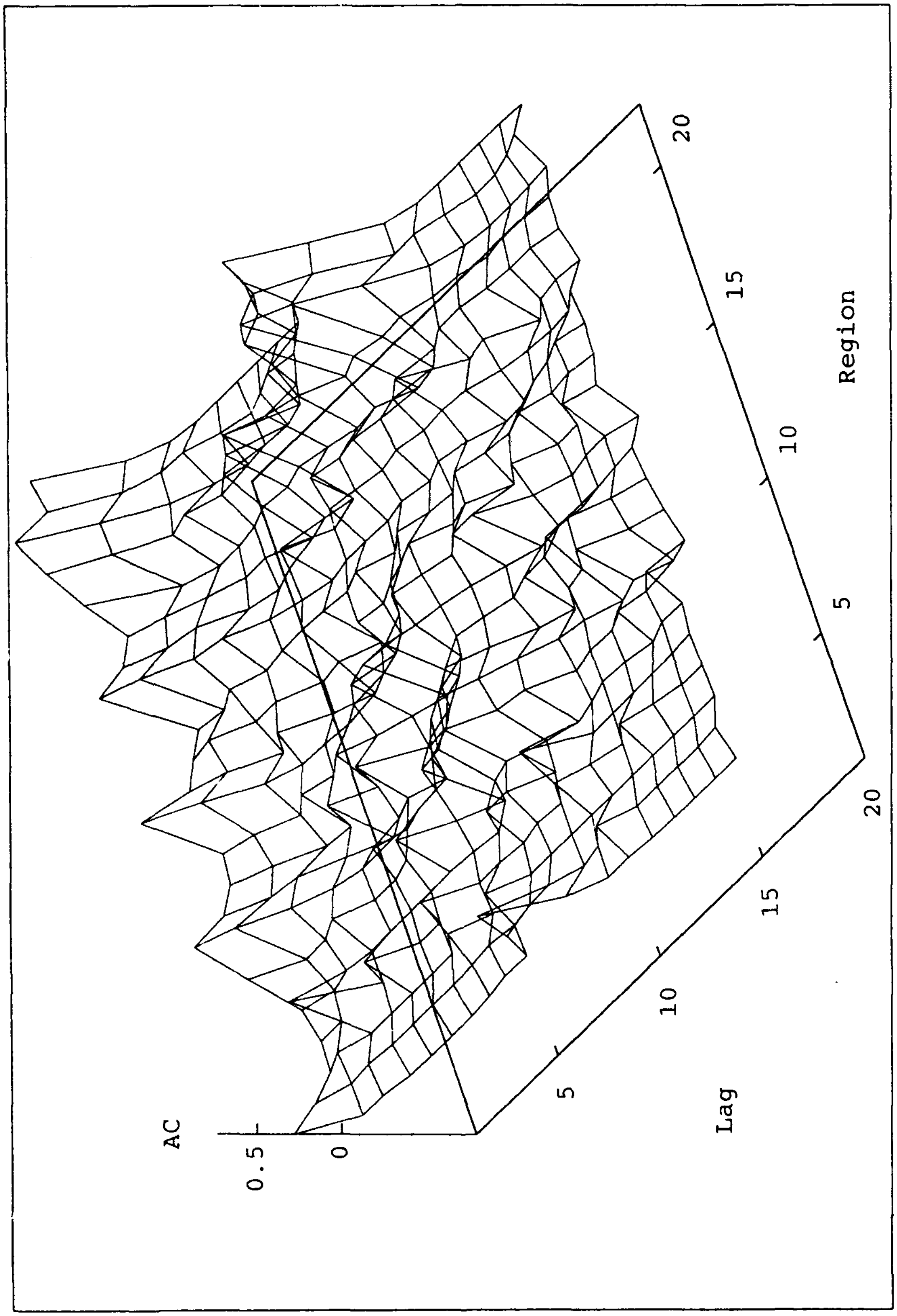


Figure 19. Autocorrelations of $NHA_{2j1}$ from Perspective B

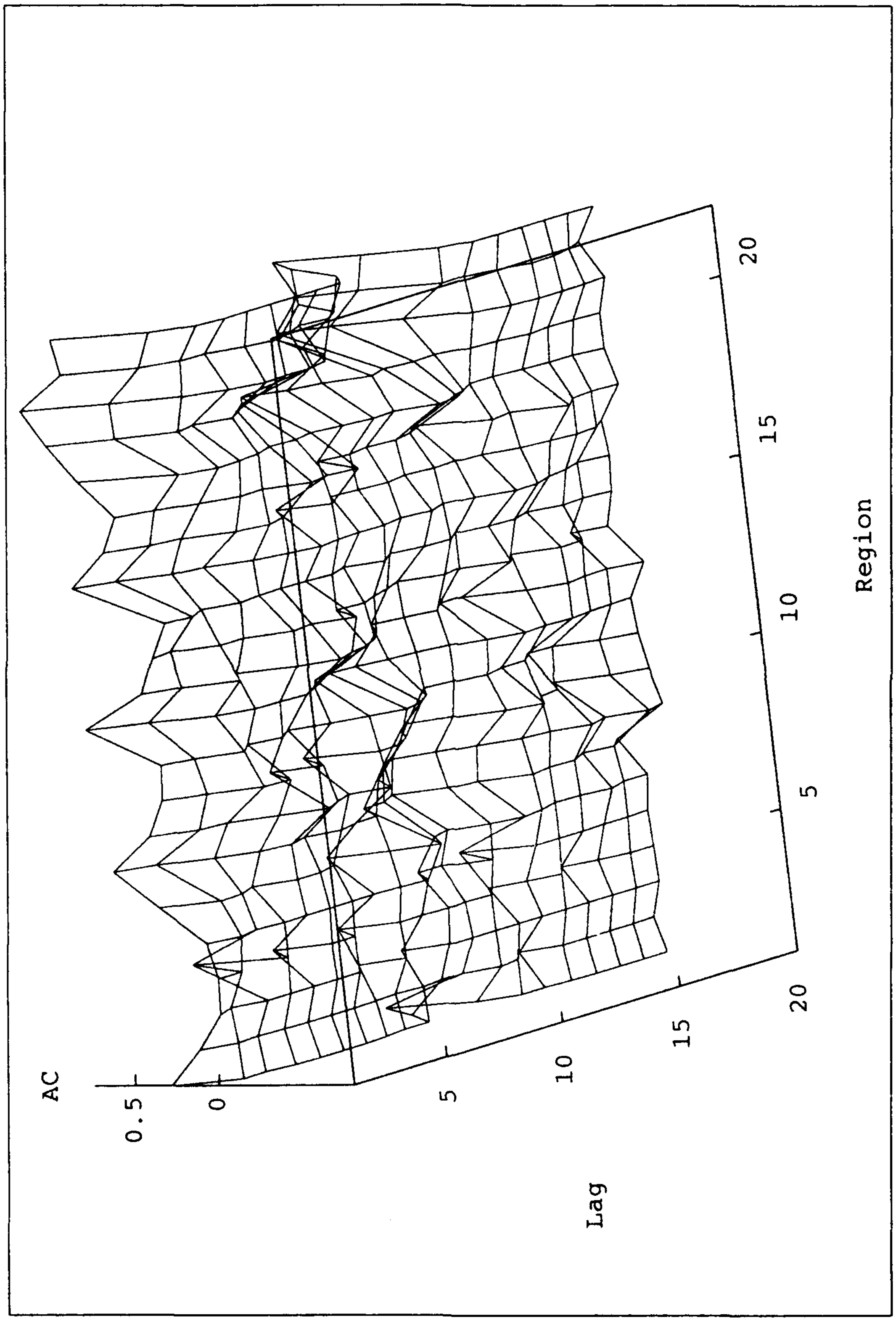


Figure 20. Autocorrelations of $NHA_{2j1}$ from Perspective C

Table 3. Significant Autocorrelations of $NHA_{2j1}$

| LAG | NUMBER OF REGIONS |
|---|---|
| 1 | 13 |
| 2 | 5 |
| 3 | 2 |
| 4 | 0 |
| 5 | 1 |
| 6 | 2 |
| 7 | 2 |
| 8 | 2 |
| 9 | 2 |
| 10 | 3 |
| 11 | 6 |
| 12 | 6 |
| 13 | 3 |
| 14 | 3 |
| 15 | 1 |
| 16 | 0 |
| 17 | 1 |
| 18 | 0 |
| 19 | 1 |
| 20 | 0 |

$$NHAS_{2j1}(t) = NHA_{2j1}(t) - NHA_{2j1}(t-12) \qquad (23)$$

where NHAS is the *N*ormalized *H*istorical frequency minus the *A*nalytical

probability de*S*easonalized

for all $j$, $j = 1, 2, \ldots, 22$ for the 79 time periods.

The data set lost 12 degrees of freedom and thus, the data set now contains 67 $(79 - 12)$ time periods and a total of 1,474 (67 x 22 = 1,474) observations. The values for $NHAS_{2,j,1}$ can be found in APPENDIX H.

### 4.7 Step 1: Data Analysis of the New Transformed Data Set $NHAS_{2j1}$

Three-dimensional plots of $NHAS_{2j1}$ from several perspectives were examined and can be found in Figure 21, Figure 22, and Figure 23. The $x$-axis is set up such that $x = 1$ corresponds to period 1, $x = 2$ corresponds to period 2,..., and $x = 67$ corresponds to period 67. $x = i$ now corresponds to period $i$, as opposed to a month, because period $i$ represents the difference between months that are 12 months apart. For example, $x = 1$ corresponds to January 1986 minus January 1985.

Differencing the data by 12 lags appears to have smoothed the data in the spatial dimension. In the temporal dimension, the 12 month season appears to still be present. This implies that the seasonality in the data is not a non-stationary seasonality. Once again, regions 7 and 11 appear to have relatively high values.

### 4.8 Step 2: Autocorrelation Analysis on the $NHAS_{2j1}$ Values

Three-dimensional plot of the autocorrelations for $NHAS_{2j1}$ from several perspectives can be found in Figure 51, Figure 52, and Figure 53. The values of the autocorrelations of $NHAS_{2,j,1}$ can be found in APPENDIX I.

Appendix J contains two-dimensional plots of the autocorrelations for each of the twenty-two geographical regions.

Table 4 lists the number of regions that had an autocorrelation value significantly different from zero at each lag. Thirteen regions have a $NHAS_{2j1}$ autocorrelation value significantlv different from zero at lag 12, resulting in the conclusion that a 12 month season is present in $NHAS_{2j1}$. $NHAS_{2j1}$ has seven more regions with an autocorrelation value significantly different from zero at lag 12 than $NHA_{2j1}$. Differencing $NHA_{2j1}$ did not remove the 12 month seasonality, resulting in the conclusion that the seasonality may be stationary.

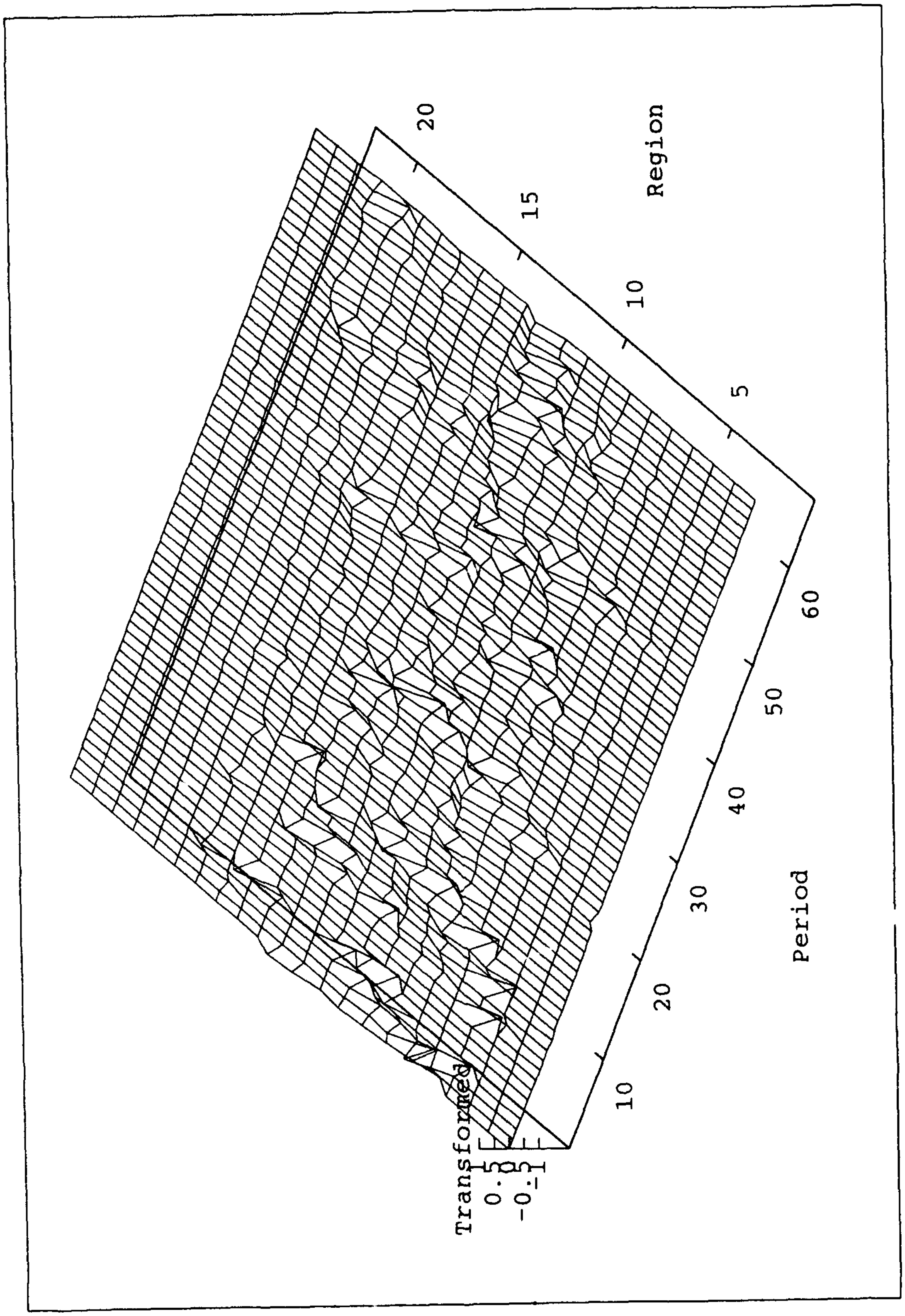


Figure 21. Normalized Historical - Analytical Data De-seasonalized $NHAS_{2j1}$ from Perspective 1

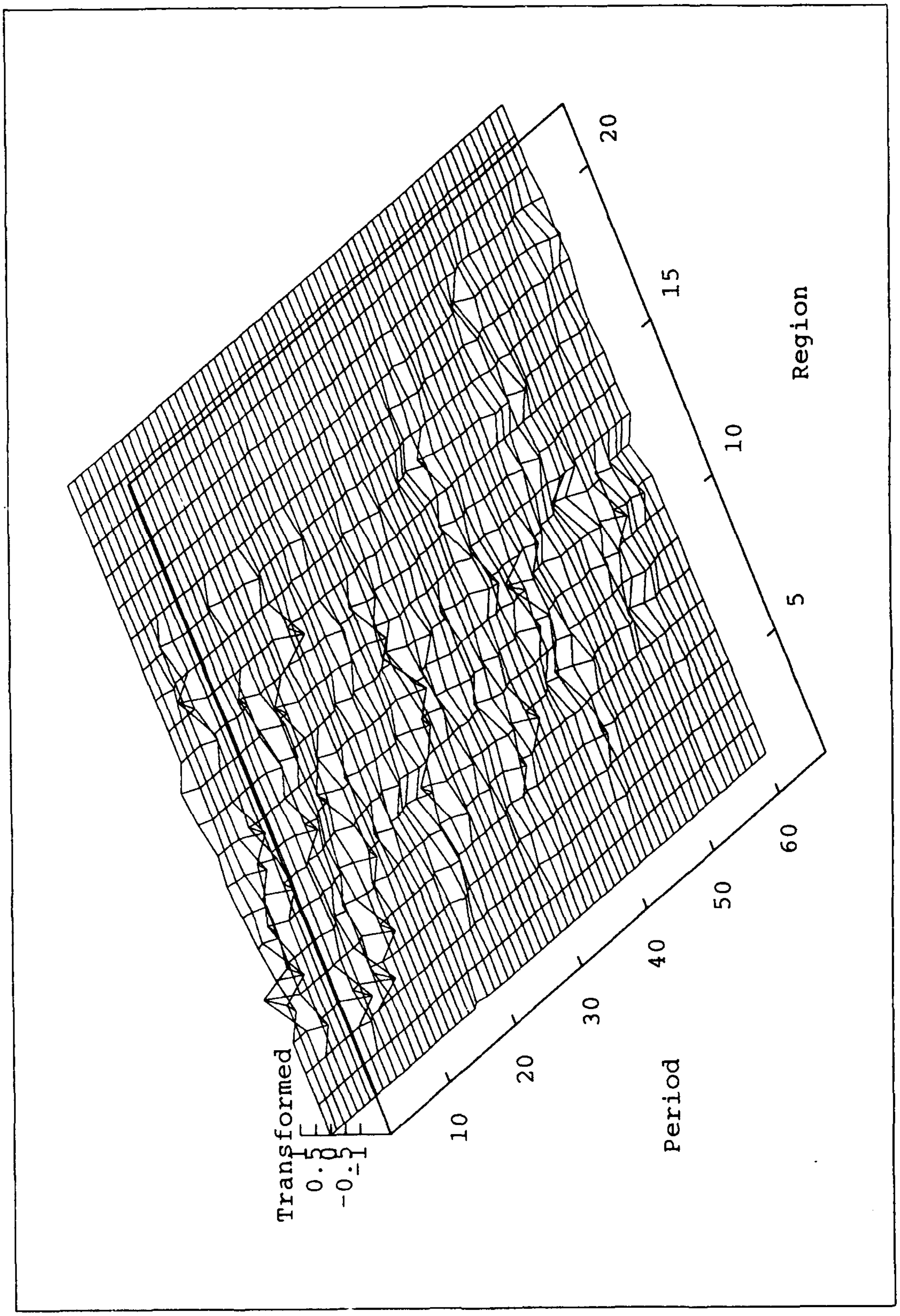


Figure 22. Normalized Historical - Analytical Data De-seasonalized $NHAS_{2j1}$ from Perspective 2

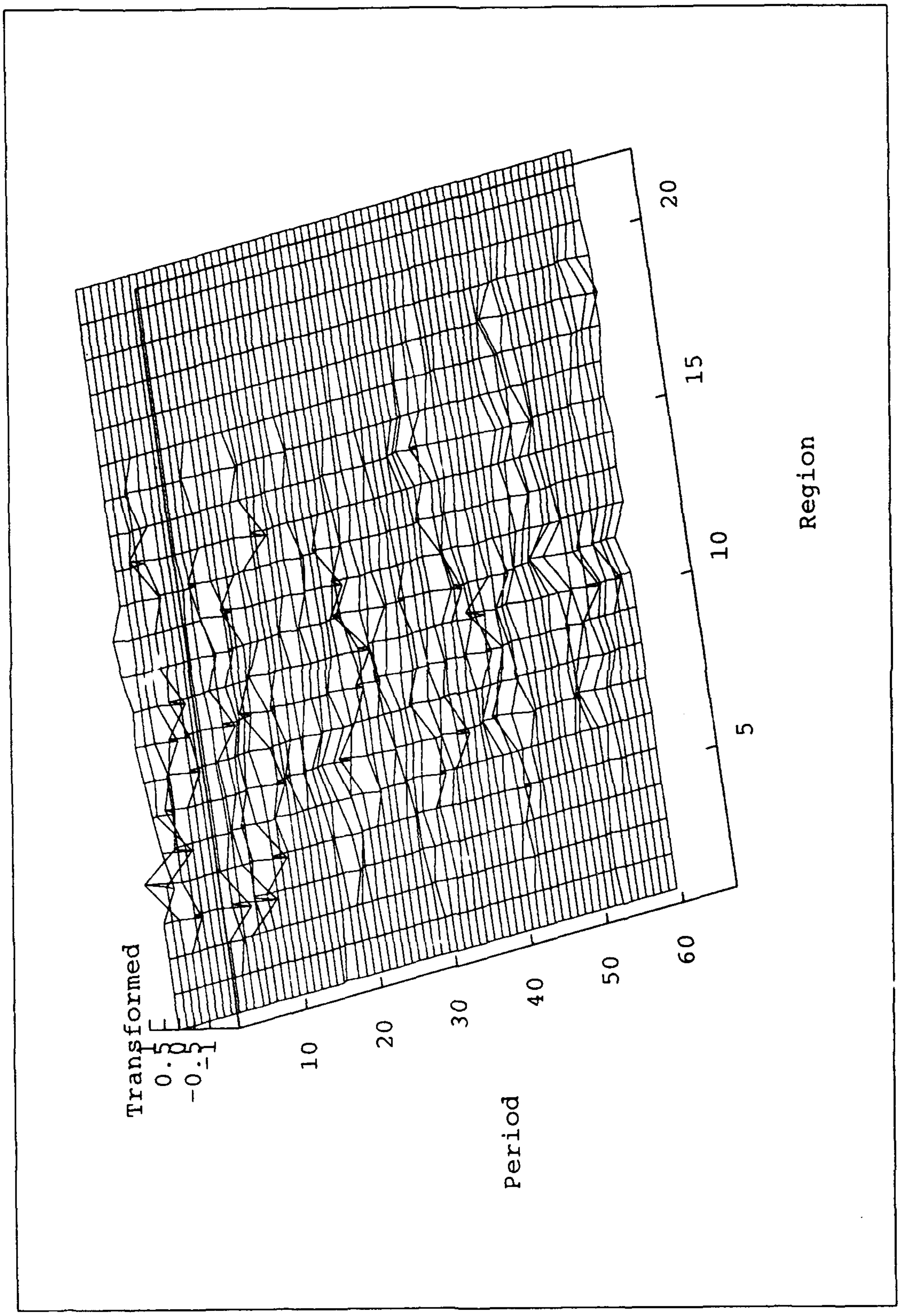


Figure 23. Normalized Historical - Analytical Data De-seasonalized $NHAS_{2j1}$ from Perspective 3

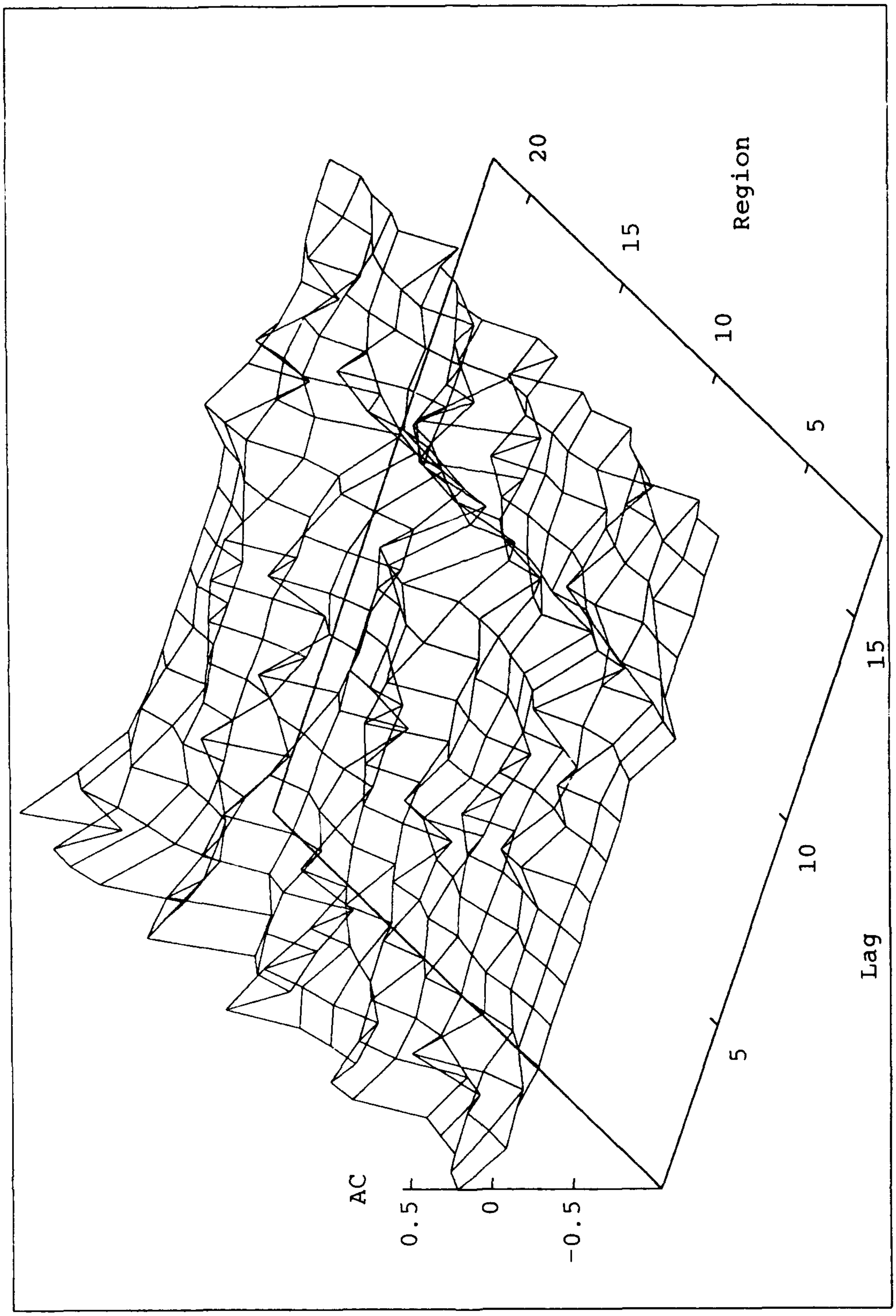


Figure 24. Autocorrelations of $NHAS_{2j1}$ from Perspective A

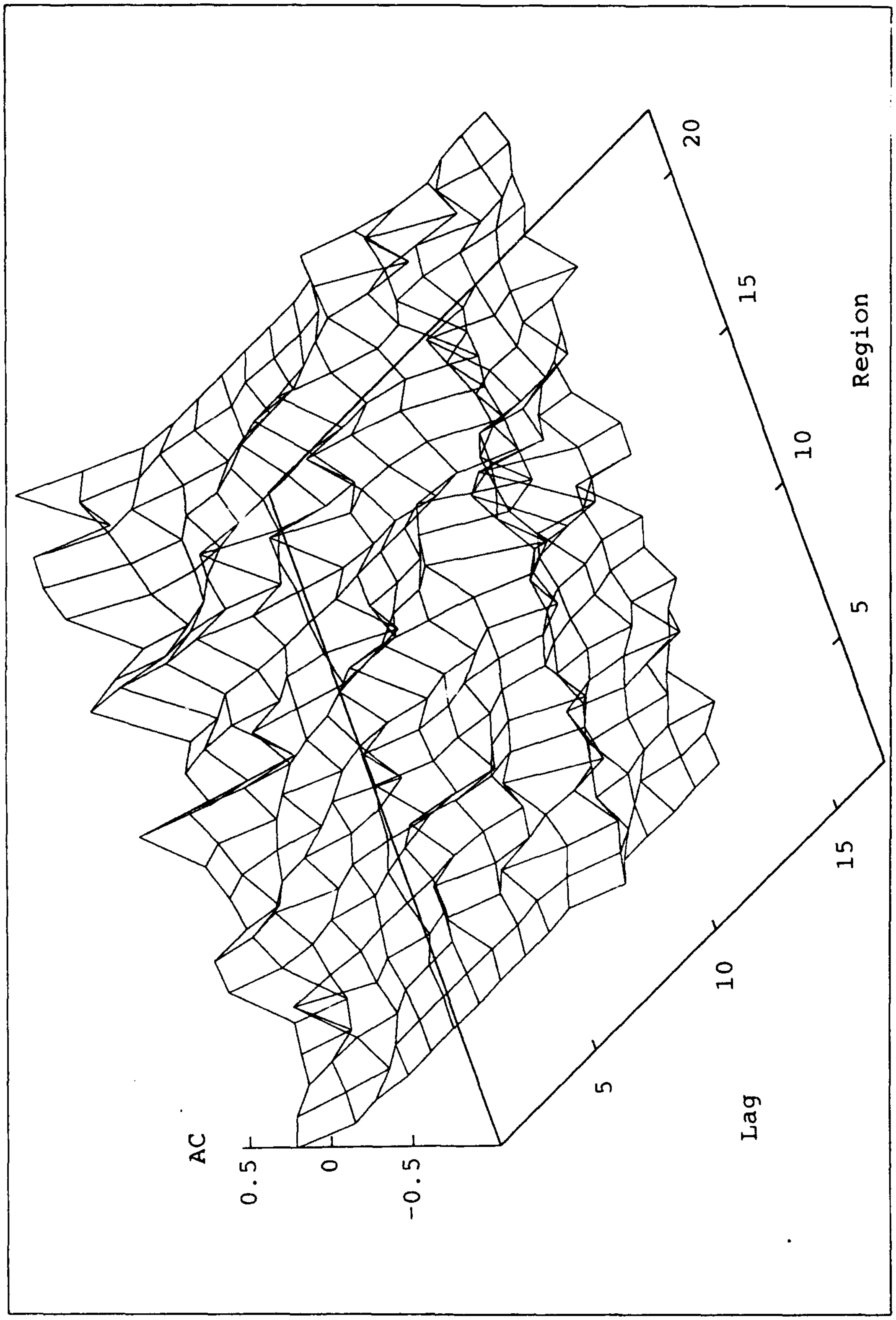


Figure 25. Autocorrelations of $NHAS_{2j1}$ from Perspective B

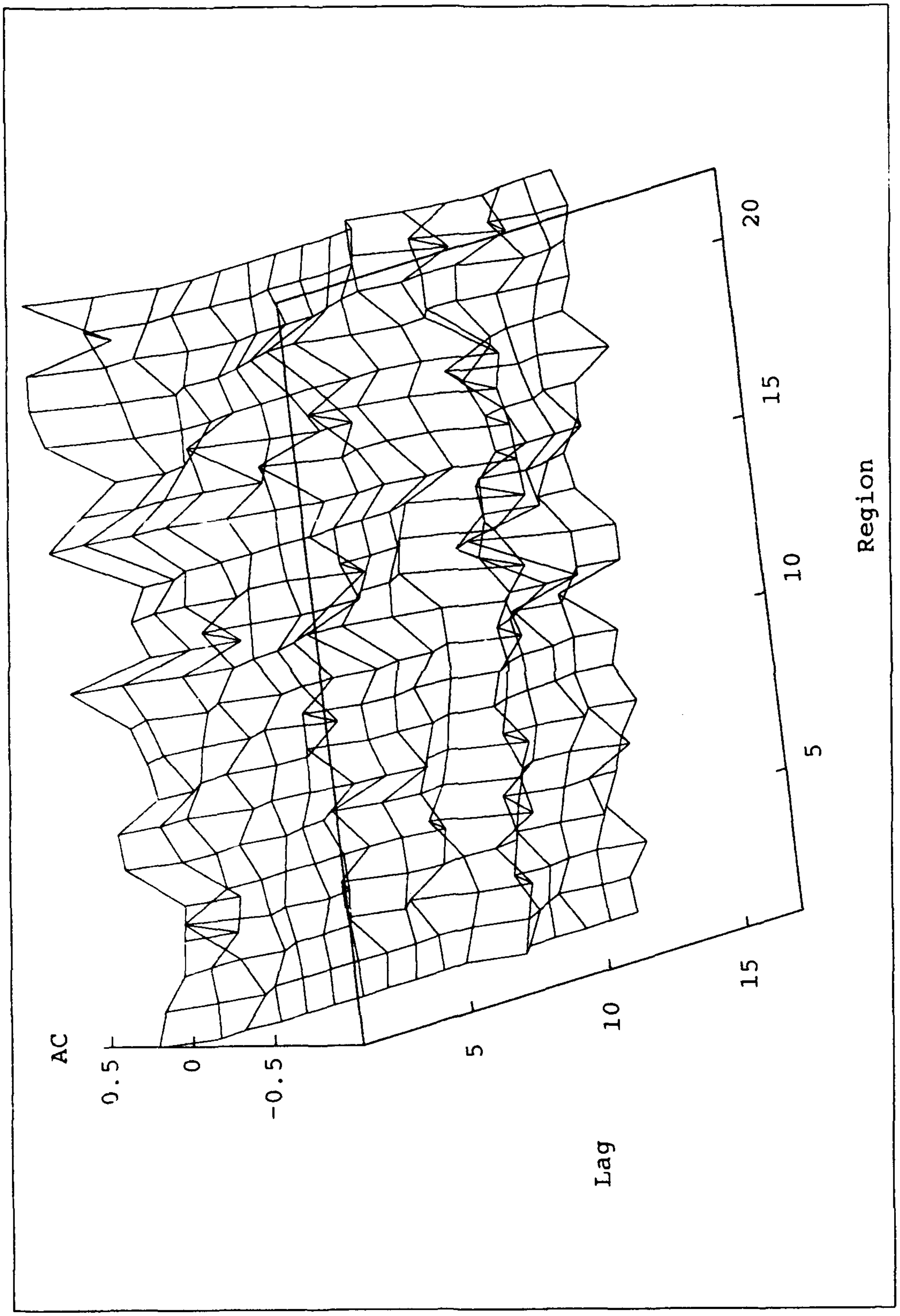


Figure 26. Autocorrelations of $NHAS_{2j1}$ from Persepctive C

Table 4. Significant Autocorrelations for $NHAS_{2j1}$

| LAG | NUMBER OF REGIONS |
|---|---|
| 1 | 9 |
| 2 | 4 |
| 3 | 2 |
| 4 | 1 |
| 5 | 1 |
| 6 | 0 |
| 7 | 1 |
| 8 | 1 |
| 9 | 2 |
| 10 | 1 |
| 11 | 1 |
| 12 | 13 |
| 13 | 1 |
| 14 | 1 |
| 15 | 2 |
| 16 | 1 |
| 17 | 1 |

### *4.9 Decision on What Data Set to Continue With*

At this point, a decision had to made on which data set was the best to continue analysis on. Is the best data set the $NHA_{2j1}$ values or the $NHAS_{2j1}$ values? Both the $NHA_{2j1}$ values and the $NHAS_{2j1}$ values are seasonal. The $NHAS_{2j1}$ values appear to be more smooth or uniform over the spatial dimension. If one compares the three-dimensional plots of the $NHA_{2j1}$ values to that of the $NHAS_{2j1}$ values, this spatial uniformity can become more evident. The $NHA_{2j1}$ values appear to have a "funnel" effect in that the regions used in the distant past appear to be more spread out over the regions than the more recent months. This "funnelling" does not appear to be present in the $NHAS_{2j1}$ values.

Uniformity in the spatial dimension is important in STARMA modelling. If the spatial dimension is not uniform over time, it becomes necessary to model this spatial non-uniformity. Thus, the model must account for the non-uniformity in the temporal dimension represented by the 12 month season and the funnelling effect in the spatial dimension. In order to keep the model as simple as possible, the decision to use the $NHAS_{2j1}$ values was made.

### *4.10 Step 2 Continued: Further Autocorrelation Analysis on the $NHAS_{2j1}$*

Further analysis is required on the autocorrelations of the $NHAS_{2j1}$ values now that the $NHAS_{2j1}$ values will be the data set to model on. Region 15 is the only region that did not pass the first ad hoc stationarity test. Region 15 did not pass the second ad hoc stationarity test, either. However, this should not cause any problems since region 15 is a first order neighbor.

A seasonal moving average and/or seasonal autoregressive term is expected due to the 12 month season. This 12-month season is evident in both the three-dimensional plots of $NHAS_{2j1}$ values and in the three-dimensional autocorrelation plots. Table 5 lists the number of regions that had a partial autocorrelation value significantly form zero at each lag.

Table 5. Significant Partial Autocorrelations for $NHAS_{2j1}$

| LAG | NUMBER OF REGIONS |
|---|---|
| 1 | 9 |
| 2 | 3 |
| 3 | 1 |
| 4 | 0 |
| 5 | 1 |
| 6 | 1 |
| 7 | 2 |
| 8 | 1 |
| 9 | 0 |
| 10 | 0 |
| 11 | 1 |
| 12 | 17 |
| 13 | 2 |
| 14 | 1 |
| 15 | 0 |
| 16 | 1 |
| 17 | 0 |

### 4.11 Step 3: Determination of Target Region, Neighbors, and Spatial Weights

Experts on the historical data base suggested the use of geographical region 11 for the target region (2). Exogenously from the historical data and the analytical estimates, the experts selected first order neighbors and second order neighbors to target region 11. The first order neighbors to target region 11 selected are listed in Table 6. The second order neighbors to target region 11 selected are listed in Table 7.

Table 6. First Order Neighbors to Target Region 11

| FIRST ORDER NEIGHBORS |
|---|
| Region 7 |
| Region 9 |
| Region 13 |
| Region 15 |
| Region 17 |
| Region 19 |

Table 7. Second Order Neighbors to Target Regir . 11

| SECOND ORDER NEIGHBORS |
|---|
| Region 1 |
| Region 2 |
| Region 3 |
| Region 4 |
| Region 5 |
| Region 6 |
| Region 8 |
| Region 10 |
| Region 12 |
| Region 14 |
| Region 16 |
| Region 18 |
| Region 20 |
| Region 21 |
| Region 22 |

It is interesting to note that all of the first order neighbors are odd numbered regions. The experts state that the odd numbered regions usually have higher relative monthly probabilities than the even numbered regions (2).

One set of spatial weights is usually selected to represent some physical property among the 22 geographical regions. In the case of the $NHAS_{2j1}$ values, a strong seasonality exists. In Section 4.2.3 *Two Dimensional Plots of the Average Relative Frequencies for Each Season*, there were four distinct average relative frequencies corresponding to each season of the year. Four sets of spatial weights were selected to represent the spatial weighting between the 22 geographical regions for each of the four seasons. The four average relative frequencies associated with each season were selected to be the spatial weights. The average relative frequencies of the first order neighbors to region 11 for each season were energy normalized so that the first order weights for each season sum to 1.0

Table 8 lists the weights for region 11 and its first order neighbors for all four seasons.

Table 8. Weights Between Region 11 and its First Order Neighbors

| Region | Winter | Spring | Summer | Fall |
|---|---|---|---|---|
| Region 7 | 0.68 | 0.33 | 0.04 | 0.44 |
| Region 9 | 0.17 | 0.12 | 0.01 | 0.01 |
| Region 13 | 0.12 | 0.10 | 0.26 | 0.08 |
| Region 15 | 0.01 | 0.15 | 0.28 | 0.05 |
| Region 17 | 0.01 | 0.05 | 0.06 | 0.04 |
| Region 19 | 0.01 | 0.09 | 0.15 | 0.05 |

The average relative frequencies of the second order neighbors to region 11 for each season were also energy normalized such that their sum is one.

Table 9 lists the weights for region 11 and its second order neighbors for all four seasons.

### *4.12 Step 4: Identification*

Because nine regions have an autocorrelation value and a partial autocorrelation value significantly different from zero at the first lag, it is highly probable that the STARMA model will have $p = 1$ and/or $q = 1$. Because four regions have an autocorrelation value significantly different from zero at the second lag and three regions have a partial autocorrelation value significantly different

Table 9. Weights Between Region 11 and its Second Order Neighbors

| Region | Winter | Spring | Summer | Fall |
|---|---|---|---|---|
| Region 1 | 0.00 | 0.00 | 0.00 | 0.00 |
| Region 2 | 0.02 | 0.00 | 0.00 | 0.00 |
| Region 3 | 0.03 | 0.02 | 0.00 | 0.04 |
| Region 4 | 0.15 | 0.09 | 0.00 | 0.18 |
| Region 5 | 0.16 | 0.18 | 0.01 | 0.03 |
| Region 6 | 0.26 | 0.09 | 0.08 | 0.15 |
| Region 8 | 0.13 | 0.16 | 0.14 | 0.13 |
| Region 10 | 0.12 | 0.22 | 0.18 | 0.28 |
| Region 12 | 0.08 | 0.11 | 0.13 | 0.15 |
| Region 14 | 0.01 | 0.01 | 0.20 | 0.01 |
| Region 16 | 0.03 | 0.06 | 0.10 | 0.02 |
| Region 18 | 0.01 | 0.06 | 0.10 | 0.02 |
| Region 20 | 0.00 | 0.00 | 0.00 | 0.00 |
| Region 21 | 0.00 | 0.00 | 0.01 | 0.00 |
| Region 22 | 0.00 | 0.00 | 0.00 | 0.00 |

from zero at the second lag, it is probable that the STARMA model will have $p = 2$ and/or $q = 2$. Because thirteen regions have an autocorrelation value significantly different from zero at the 12th lag and 17 regions have a partial autocorrelation value significantly different from zero at the 12th lag, it is highly probable that the STARMA model will have $sar = 12$ and/or $sma = 12$. All other $p$, $q$, $sar$, and $sma$ values appear unlikely.

Identification of a STARMA model is usually accomplished by examination of the autocorrelations and the partial autocorrelations to see which of the two has a tendency to exponentially decrease and which has a tendency to go to zero fast. Neither the autocorrelations or the partial autocorrelations exhibit a tendency to exponentially decrease to zero or go to zero fast. As a result, there is no identification of a STAR, STMA, or STARMA model. The identification is limited to $p \leq 2$ and $q \leq 2$, $sar = 12$, and $sma = 12$.

#### 4.12.1 ARMA Model Building of Target Region 11 $NHAS_{2,11,1}$ to Identify p and q.

##### 4.12.1.1 Data Analysis of Target Region 11 $NHAS_{2,11,1}$.

A two-dimensional plot of $NHAS_{2j1}$ at geographical region 11 can be found in Figure 27. The $x$-axis is set up such that $x =$

1 corresponds to observation 1, $x = 2$ corresponds to observation 2, ..., and $x = 67$ corresponds to observation 67.

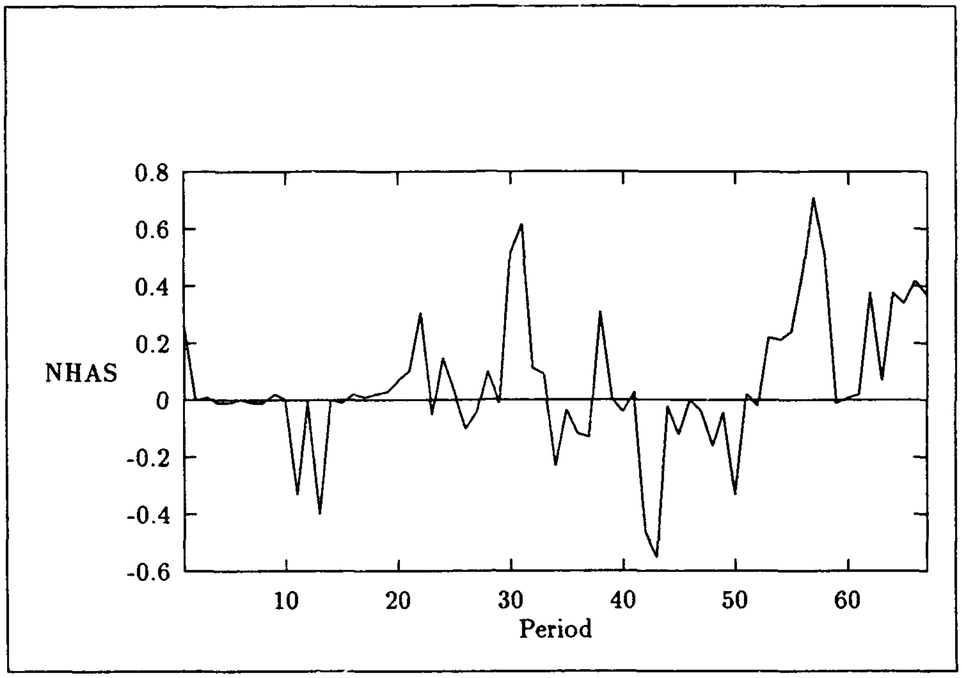


Figure 27. Geographical Region 11 $NHAS_{2,11,1}$

From analyzing the plot of region 11, there appears to be no trend in the data. Thus, the target region data is expected to be stationary. A 12 month season is not apparent from the plot of geographical region 11 except that the third block of 12 months ($x = 25$ through $x = 36$) appears similar to the fifth block of 12 months ($x = 49$ through $x = 60$). The first 12 month block ($x = 1$ through $x = 12$) and the second 12 month block ($x = 13$ through $x = 24$) appear relatively flat except for one spike at the beginning of each block and one spike at the end.

*4.12.1.2 Autocorrelation Analysis of Target Region 11 $NHAS_{2,11,1}$.* Figure 28 is a two-dimensional plot of the autocorrelations of $NHAS_{2j1}$ at Target Region 11. The $x$-axis is set up such that $x = 1$ corresponds to lag 1, $x = 2$ corresponds to lag 2,..., and $x = 17$ corresponds to lag 17.

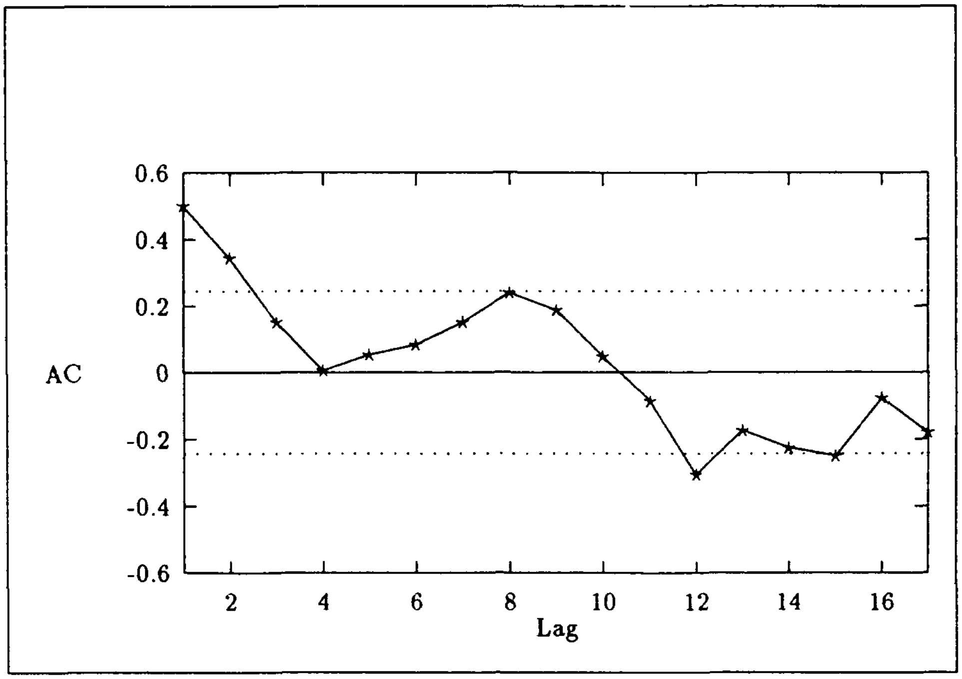


Figure 28. Autocorrelations of Target Region 11 $NHAS_{2,11,1}$

Figure 29 is a two-dimensional plot of the partial autocorrelations for Target Region 11. The $x$-axis is set up such that $x$ =1 corresponds to lag 1, $x$ = 2 corresponds to lag 2, ... , and $x$ = 17 corresponds to lag 17. The values of the partial autocorrelations for region 11 can be found in APPENDIX K.

It was determined in section *Step 2: Autocorrelation Analysis of the $NHAS_{2j1}$ Values* that the data for region 11 was stationary by passing the second stationarity test. Both the autocorrelations and the partial autocorrelations of $NHAS_{2j1}$ at target region 11 are significant at the lag 12, implying a 12 month season.

*4.12.1.3 Identification for Target Region 11 $NHAS_{2,11,1}$.* The autocorrelations are significantly different from zero at lags 1, 2, 12, and 15. The partial autocorrelations are significantly different from zero at lags 1 and 12. This would imply the possibility of $p \leq 2$ and $q \leq 2$ and a seasonal term of 12 time periods. Neither the autocorrelations and the partial autocorrelations

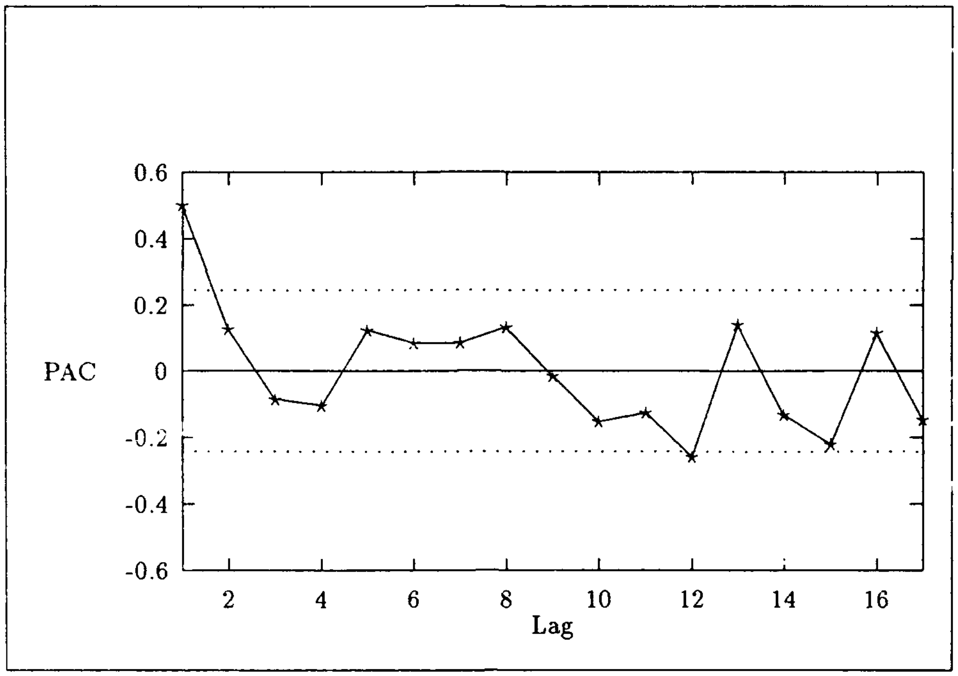


Figure 29. Partial Autocorrelations of Target Region 11 $NHAS_{2,11,1}$

appear to go to zero fast or exponentially decrease to zero. Thus, it is not possible to identify whether the model will be an AR, MA, or ARMA model. Because a 12 month season is quite apparent, a seasonal model is identified. Identified models are all SAR, SMA, and SARMA models with all combinations of $p \leq 2$, $q \leq 2$, $sar = 12$, and $sma = 12$.

*4.12.1.4 Estimation for Target Region 11 $NHAS_{2,11,1}$.* No model specification was identified because the autocorrelations and the partial autocorrelations did not appear to either go to zero fast or exponentially go to zero. As a result, the 22 combinations of $p \leq 2$, $q \leq 2$, $sar = 12$, and $sma = 12$ were all estimated.

*4.12.1.5 Diagnostic Checking for Target Region 11 $NHAS_{2,11,1}$.* Table 10 summarizes the significance of the parameter coefficients in each of the 22 estimated models. A 'S' in the AR column means that the coefficient of the highest AR order in the corresponding estimated model is significantly different from zero at the 90% confidence level. Likewise, a 'S' in the MA column means that the coefficient of the highest MA order in the corresponding estimated model is significantly different from zero with 90% confidence. A 'S' in the SAR column means that the coefficient of the SAR(12) term in the corresponding estimated model is significantly different from zero at the 90% confidence level. Likewise, a 'S' in the SMA column means that the coefficient of the SMA(12) term in the corresponding estimated model is significantly different from zero with 90% confidence.

A 'NS' in the AR column means that the coefficient of the highest AR order in the corresponding estimated model is not significantly different from zero at the 90% confidence level. Likewise, a 'NS' in the MA column means that the coefficient of the highest MA order in the corresponding estimated model is significantly different from zero with 90% confidence. A 'NS' in the SAR column means that the coefficient of the SAR(12) term in the corresponding estimated model is not significantly different from zero at the 90% confidence level. Likewise, a 'NS' in the SMA column means that the coefficient of the SMA(12) term in the corresponding estimated model is not significantly different from zero with 90% confidence.

A blank space in any column specifies that the coefficient was not applicable.

From Table 10, 16 of the 22 estimated models can be thrown out for one of the following reasons: the coefficient of the highest order of $p$ was insignificant, the coefficient of the highest order of $q$ was insignificant, the coefficient of $sar(12)$ was insignificant, or the coefficient of $sma(12)$ was insignificant.

*4.12.1.6 Forecasting for Target Region 11 $NHAS_{2,11,1}$.* Six of the estimated models passed the significance portion of diagnostic checking. Table 11 summarizes the $\bar{R}^2$ value and the SSR for each of the six estimated models that passed the significance portion of diagnostic checking.

where $\bar{R}^2$ is the adjusted $R^2$ and

SSR is the sum of the squared residuals.

Table 10. Significance of the Coefficients for All Estimated ARMA Models for Target Region 11 $NHAS_{2j1}$

| Model | AR | MA | SAR | SMA |
|---|---|---|---|---|
| MA(1) | | S | | |
| AR(1) | S | | | |
| $MA(1)_{12}$ | | S | | S |
| $AR(1)_{12}$ | S | | S | |
| ARMA(1,1) | S | NS | | |
| $ARMA(1,1)_{0,12}$ | S | NS | | S |
| $ARMA(1,1)_{12,0}$ | S | NS | S | |
| $ARMA(1,1)_{12,12}$ | S | NS | NS | S |
| MA(2) | | S | | |
| $SMA(2)_{12}$ | | S | | S |
| AR(2) | NS | | | |
| $SAR(2)_{12}$ | NS | | S | |
| ARMA(2,1) | NS | NS | | |
| $ARMA(2,1)_{12,0}$ | NS | NS | S | |
| $ARMA(2,1)_{0,12}$ | NS | NS | | S |
| ARMA(1,2) | NS | NS | | |
| $ARMA(1,2)_{12,0}$ | NS | NS | S | |
| $ARMA(1,2)_{0,12}$ | S | NS | | S |
| ARMA(2,2) | NS | NS | | |
| $ARMA(2,2)_{12,0}$ | NS | NS | S | |
| $ARMA(2,2)_{0,12}$ | NS | NS | | S |
| $ARMA(2,2)_{12,12}$ | NS | NS | NS | S |

Table 11. Significant ARMA Models for Target Region 11 $NHAS_{2,11,1}$

| MODEL | $R^2$ | SSR |
|---|---|---|
| MA(1) | 0.164 | 2.990 |
| AR(1) | 0.249 | 2.654 |
| $MA(1)_{12}$ | 0.339 | 2.329 |
| $AR(1)_{12}$ | 0.353 | 1.969 |
| MA(2) | 0.239 | 2.682 |
| $SMA(2)_{12}$ | 0.407 | 2.058 |

In order to throw out more of the model combinations, the $\bar{R}^2$ and SSR were examined. The criterion selected is to maximize the $\bar{R}^2$ while minimizing the SSR. The SMA$(2)_{12}$ model, SAR$(1)_{12}$ model, and the SMA$(1)_{12}$ model have the largest $\bar{R}^2$ value. The SAR$(1)_{12}$, SMA$(2)_{12}$ model and the SMA$(1)_{12}$ model have the smallest SSR. The value of $\bar{R}^2$ for the SMA$(2)_{12}$ model is 13.3% better than that of the SAR$(1)_{12}$ model, and 16.7% better than that of the SMA$(1)_{12}$. The value of $\bar{R}^2$ for the SAR$(1)_{12}$ model is 4.0% better than that of the SMA$(1)_{12}$ model. The value of SSR for the SAR$(1)_{12}$ model is 4.3% better than that of the SMA$(2)_{12}$ model and is 15.5% better than that of the SMA$(1)_{12}$ model. The SSR value of the SMA$(2)_{12}$ model is 1.0% better than that of the SMA$(1)_{12}$ model.

The SMA$(2)_{12}$ model is clearly superior in terms of maximizing $\bar{R}^2$ since the value of its $\bar{R}^2$ is over 13% better than all of the other estimated models that past the significance portion of diagnostic checking. The AR$(1)_{12}$ model and the SMA$(2)_{12}$ have SSR values that are less than 5% different from each other. Both the AR$(1)_{12}$ model and the SMA$(2)_{12}$ model have an SSR value that is less than 5% smaller than that of the MA$(1)_{12}$ model.

Because the SMA$(2)_{12}$ model has the superior $\bar{R}^2$ value and has a SSR value that is less than 5% different from the AR$(1)_{12}$ model, the best model to forecast target region 11 is the SMA$(2)_{12}$ model specification.

Table 12 lists the coefficients of the terms in the SMA$(2)_{12}$ model along with the standard error of the coefficients, the $t$-statistic of the coefficients, and the two tailed significance of the coefficients, all rounded to the nearest one-thousandths.

Table 12. SMA$(2)_{12}$ model Designation for Target Region 11 $NHAS_{2,11,1}$

| Variable | Coefficient | Std. Error | T-STAT | 2-Tail Sig. |
|---|---|---|---|---|
| C | 0.049 | 0.022 | 2.215 | 0.030 |
| MA(1) | 0.434 | 0.112 | 3.890 | 0.000 |
| MA(2) | 0.423 | 0.111 | 3.811 | 0.000 |
| SMA(12) | -0.067 | 0.139 | -4.808 | 0.000 |

*4.12.2 ARMA Model Building of Combined Series to Identify $\lambda_p$ and $m_q$.* ARMA model building to develop the spatial relationship was conducted on the series of data that contains the combination of the target region values, the first order values, and the second order values. The combination series contains 201 observations (67 x 3 = 201). Because the model selected as the best model for target region 11 was a SMA model, the ARMA model building to develop the spatial relationship is limited to MA models. Because there are only first and second orders to the data and the best model for target region 11 was a SMA model, the ARMA model building to develop the spatial relationship is limited to $q$ values less than or equal to 2.

*4.12.2.1 Data Analysis of Combined Series.* Figure 30 is a two-dimensional plot of the combined series. The $x$ axis is set up such that $x$ = 1 corresponds to observation 1 of the Target Region 11 series, $x$ = 2 corresponds to observation 1 of the weighted sum of the first order neighbors series, $x$ = 3 corresponds to observation 1 of the weighted sum of the second order neighbors series, $x$ = 4 corresponds to observation 2 of the Target Region 11 series, $x$ = 5 corresponds to observation 2 of the first order neighbors series, $x$ = 6 corresponds to observation 2 of the second order neighbors series, . . . , $x$ = 199 corresponds to observation 67 of the Target Region 11 series, $x$ = 200 corresponds to observation 67 of the first order neighbors series, and $x$ = 201 corresponds to observation 67 of the second order neighbors series. The values of the combined series can be found in APPENDIX 10.

The plot of the combined series does not appear to show any trend. There is no need to examine the combined series for seasonality since the only thing to be determined is the spatial relationship.

*4.12.2.2 Autocorrelation Analysis of Combined Series.* Because this series is a combination of the three series, stationarity is not necessary. However, examination of the autocorrelations and partial autocorrelations is important to find significant autocorrelations and significant partial autocorrelations. Because there are 201 total observations in the combined series, it is rec-

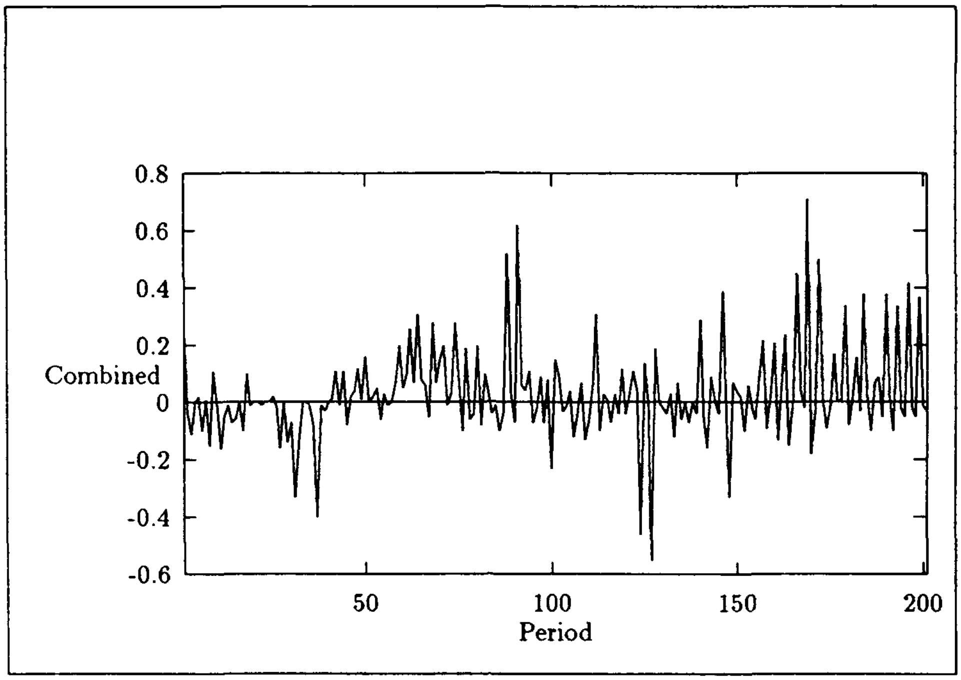


Figure 30. Combined Series of $NHAS_{2,j,1}$ with Target Region 11

ommended to calculate the autocorrelations and the partial autocorrelations up to lag 51 ($\frac{201}{4}$ = 50.25). However, *MicroTSP* is limited to the calculation of autocorrelations and partial autocorrelations up to 44 lags. Thus, only the autocorrelations and the partial autocorrelations up to lag 44 were calculated. Figure 31 is a two-dimensional plot of the autocorrelations for the combined series. The $x$-axis is set up such that $x = 1$ corresponds to lag 1, $x = 2$ corresponds to lag 2, ..., and $x = 44$ corresponds to lag 44. The values of the autocorrelations of the combined series can be found in APPENDIX M.

Figure 32 is a two-dimensional plot of the partial autocorrelations for the combined series. The $x$-axis is set up such that $x = 1$ corresponds to lag 1, $x = 2$ corresponds to lag 2,..., and $x = 44$ corresponds to lag 44. The values of the partial autocorrelations for the combined series can be found in APPENDIX M.

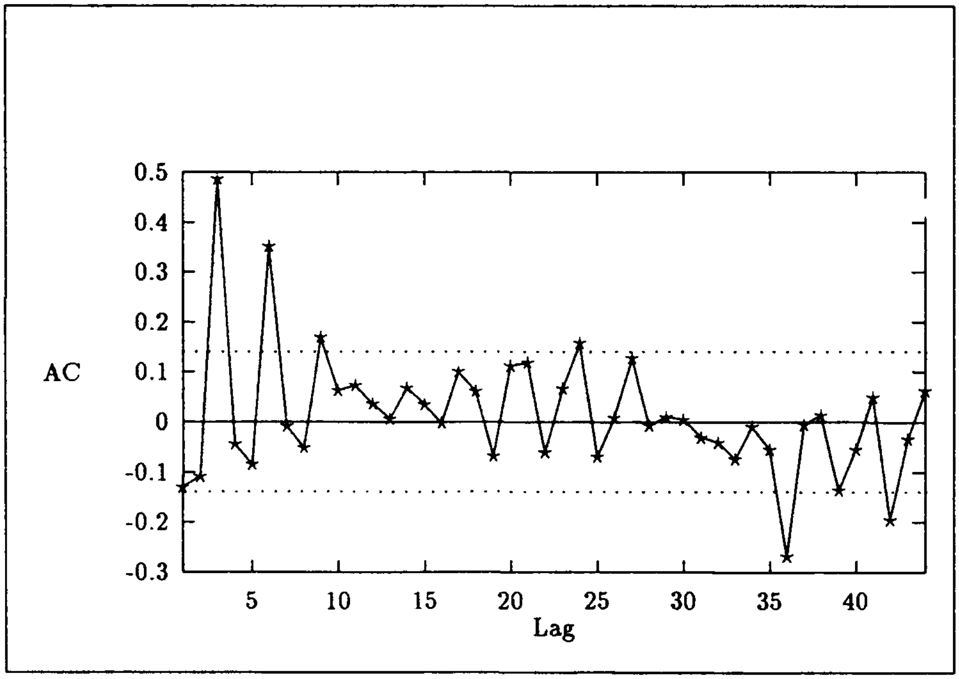


Figure 31. Autocorrelations of the Combined Series

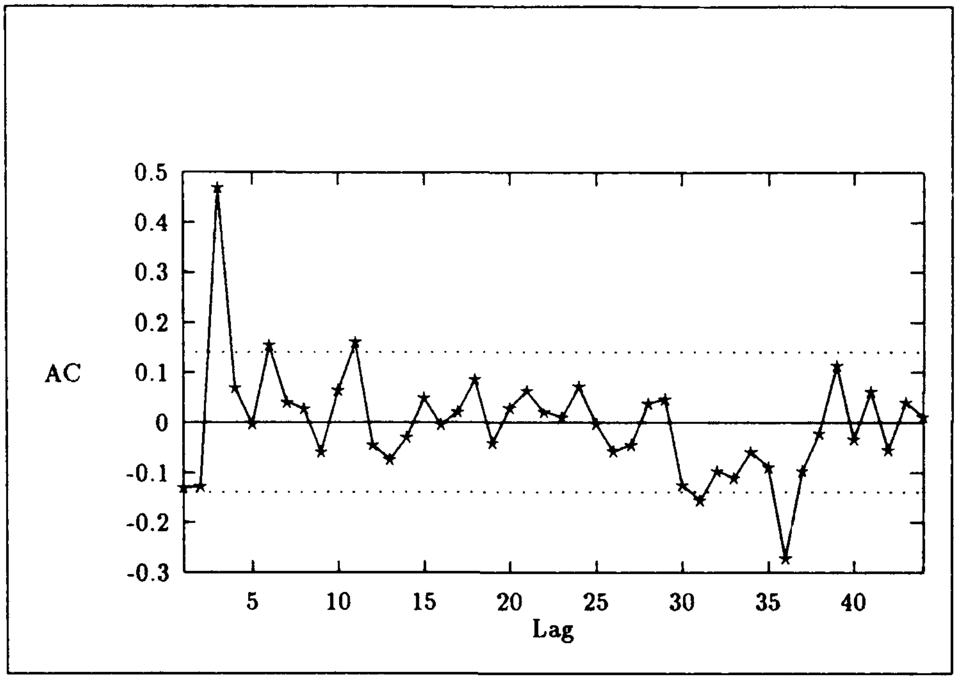


Figure 32. Partial Autocorrelations of the Combined Series

The autocorrelations are significantly different from zero at lags 3, 6, 9, 24, 36, and 42. The partial autocorrelations are significantly different from zero at lags 3, 6, 24, 36. All of the lags that exhibit either an autocorrelation or a partial autocorrelation significantly different from zero occur at multiples of 3. This is expected since the series was combined from three separate series. The significant autocorrelation and partial autocorrelation at lag 36 corresponds to the significant autocorrelation and partial autocorrelation at lag 12 for target region 11.

*4.12.2.3 Identification for the Combined Series.* No autocorrelation or partial autocorrelation at lag 1 or lag 2 was significantly different from zero. Despite this fact, the following models are identified: MA(1) and MA(2).

*4.12.2.4 Estimation for the Combined Series.* Both identified models were estimated using *MicroTSP.*

*4.12.2.5 Diagnostic Checking for the Combined Series.* Table 13 summarizes the significance of the parameter coefficient(s) of the estimated MA(1) and MA(2) model. A "S' in the MA column means that the coefficient of the highest MA order in the correspoding estimated model is significantly different from zero with 90% confidence. A 'NS' in the MA column means that the coefficient of the highest MA order in the correspoding estimated model is significantly different from zero with 90% confidence.

Table 13. Significance of the Coefficients for All Estimated ARMA Models for the Combined Series

| Model | MA |
|---|---|
| MA(1) | S |
| MA(2) | NS |

The coefficient for the MA(2) term in the MA(2) model was not significantly different from zero and the 90% confidence level. The MA(2) failed to pass diagnostic checking and as a result, the MA(2) model is removed from consideration.

*4.12.2.6 Forecasting for the Combined Series.* The $\bar{R}^2$ value for the MA(1) model on the combined series is 0.014. The SSR value for the MA(1) model on the combined series is 5.076. Table 14 lists the coefficients of the terms in the MA(1) model along with the standard error of the coefficients, the $t$-statistic of the coefficients, and the two tailed significance of the coefficients, all rounded to the nearest one-thousandths.

Table 14. MA(1) Model for the Combined Series

| Variable | Coefficient | Std. Error | T-STAT | 2-Tail Sig. |
|---|---|---|---|---|
| C | 0.029 | 0.011 | 2.594 | 0.009 |
| MA(1) | -0.138 | 0.0709 | -1.951 | 0.051 |

The two tailed significance values show that the coefficients for the terms in the MA(1) model are significant at the 90% confidence level.

The MA(1) model is the best and most parsimonious model to represent the spatial relationship between the target region, the first order terms, and the second order terms.

*4.12.3 Summary of Identification.* The purpose of building an ARMA model on the target region was to identify $p$, $q$, $sar$, and $sma$ for the STARMA model. The best ARMA model for target region 11 was SMA$(2)_{12}$ resulting in an identified $p = 0$, $q = 2$, $sar = 0$, and $sma = 12$.

The purpose of building an ARMA model on the combined series was to determine the spatial relationships between the target region and its neighbors. The MA(1) model was the best and most parsimonious model of the combined time series. The $q$ value of the combined time series corresponds to the $m_k$ value identified for the STARMA model. The value of $m_k = 1$ is identified for the STARMA model. The identified STARMA model is SSTMA$(2_{1,1})_{12}$.

### *4.13 Step 5: Estimation*

*MicroTSP* was used to estimate the identified SSTMA models.

### *4.14 Step 6: Diagnostic Checking*

Table 15 lists the coefficients of the terms in the SSTMA$(2_{1,1})_{12}$ model along with the standard error of the coefficients, the $t$-statistic of the coefficients, and the two tailed significance of the coefficients, all rounded to the nearest one-thousandths.

Table 15. The SSTMA$(2_{1,1})_{12}$ Model

| Parameter | Coefficient | Std. Error | T-STAT | 2-Tail Sig. |
|---|---|---|---|---|
| C | 0.027 | 0.009 | 3.031 | 0.002 |
| $\theta_{1,0}$ | 0.424 | 0.064 | 6.594 | 0.000 |
| $\theta_{1,1}$ | -0.160 | 0.064 | -2.498 | 0.012 |
| $\theta_{2,0}$ | 0.396 | 0.064 | 6.190 | 0.000 |
| $\theta_{2,1}$ | -0.131 | 0.064 | -2.037 | 0.042 |
| $sma(12)$ | -0.586 | 0.078 | -7.486 | 0.000 |

The two tailed significance shows that all of the coefficients for the SSTMA$(2_{1,1})_{12}$ model are significantly different from zero at the 90% confidence level. The SSTMA$(2_{1,1})_{12}$ passed the significance portion of diagnostic checking.

Figure 33 is a two-dimensional plot of the residuals from fitting the SSTMA$(2_{1,1})_{12}$ model to the combined series. The $x$-axis is set up such that $x = 1$ corresponds to observation 1, $x = 2$ corresponds to observation 2, ..., and $x = 201$ corresponds to observation 201. The values of the residuals can be found in APPENDIX N.

Figure 34 is a two-dimensional plot of the residual autocorrelations from fitting the SSTMA$(2_{1,1})_{12}$ model to the combined series. The $x$-axis is set up such that $x = 1$ corresponds to lag 1, $x = 2$ corresponds to lag 2, ..., and $x = 44$ corresponds to lag 44. Only the autocorrelations at the first 44 lags were calculated, as opposed to the first 51 ($\frac{201}{4} = 50.25$) lags, because *MicroTSP* is limited to

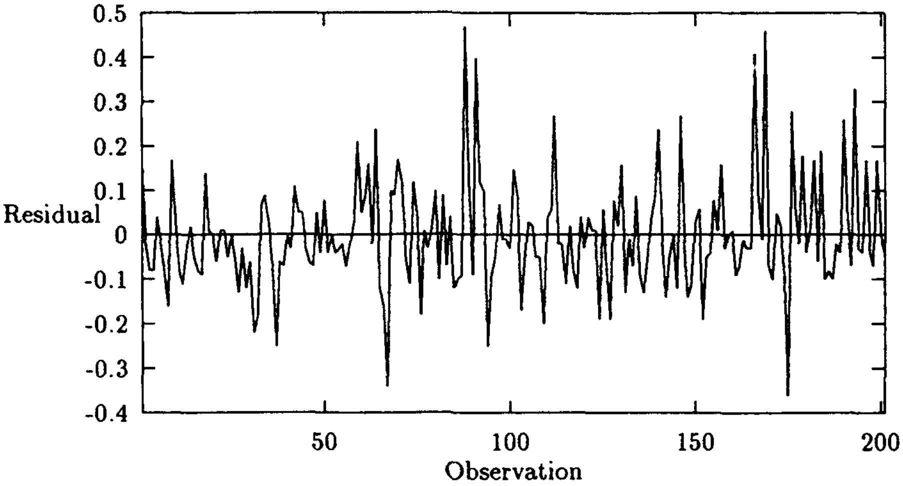


Figure 33. Residuals from Fitting the SSTMA($2_{1,1}$)$_{12}$ Model to the Combined Series of Target Region 11

calculating autocorrelation values at the first 44 lags. The values of the residual autocorrelations can be found in APPENDIX O.

Figure 35 is a two-dimensional plot of the residual partial autocorrelations from fitting the SSTMA($2_{1,1}$)$_{12}$ model to the combined series. The $x$-axis is set up such that $x = 1$ corresponds to lag 1, $x = 2$ corresponds to lag 2, ..., and $x = 44$ corresponds to lag 44. Only the partial autocorrelations at the first 44 lags were calculated, as opposed to the first 51 ($\frac{201}{4} = 50.25$) lags, because *MicroTSP* is limited to calculating partial autocorrelation values at the first 44 lags. The values of the residual partial autocorrelations can be found in APPENDIX O.

From examining Figure 33 of the residuals, there does not appear to be any trend or pattern in the residuals. The $Q$-statistic of 27.885 falls below the $Q$-critical value indicating the residuals are stationary. Only one autocorrelation value was significantly different from zero. This significant autocorrelation value occurred at the 9th lag. None of the partial autocorrelations were significantly

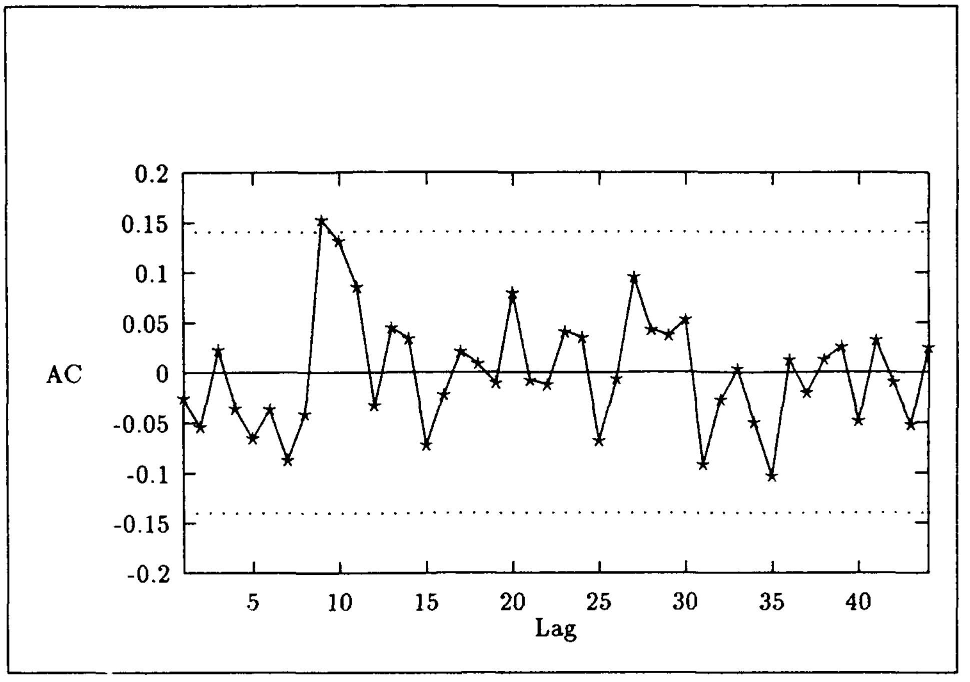


Figure 34. Residual Autocorrelations from Fitting the SSTMA$(2_{1,1})_{12}$ Model to the Combined Series of Target Region 11

different from zero. The SSTMA$(2_{1,1})_{12}$ model passed the white noise residuals portion of diagnostic checking.

The SSTMA$(2_{1,1})_{12}$ model passed both portions of the diagnostic checking and thus, the SSTMA$(2_{1,1})_{12}$ model adequately describes and represents the combined series.

### *4.15 Step 7: Forecasting*

The SSTMA$(2_{1,1})_{12}$ model has a $\bar{R}^2$ value of 0.382 and a SSR value of 3.115. The $\bar{R}^2$ value can be interpreted as the following: 38.2% of the variance in the combined series is explained by the model.

The SSTMA$(2_{1,1})_{12}$ model is the best and most parsimonious model to forecast the combined series. As a result, SSTMA$(2_{1,1})_{12}$ is the best and most parsimonious model to forecast the values of $NHAS_{2,j,1}$ for target region 11.

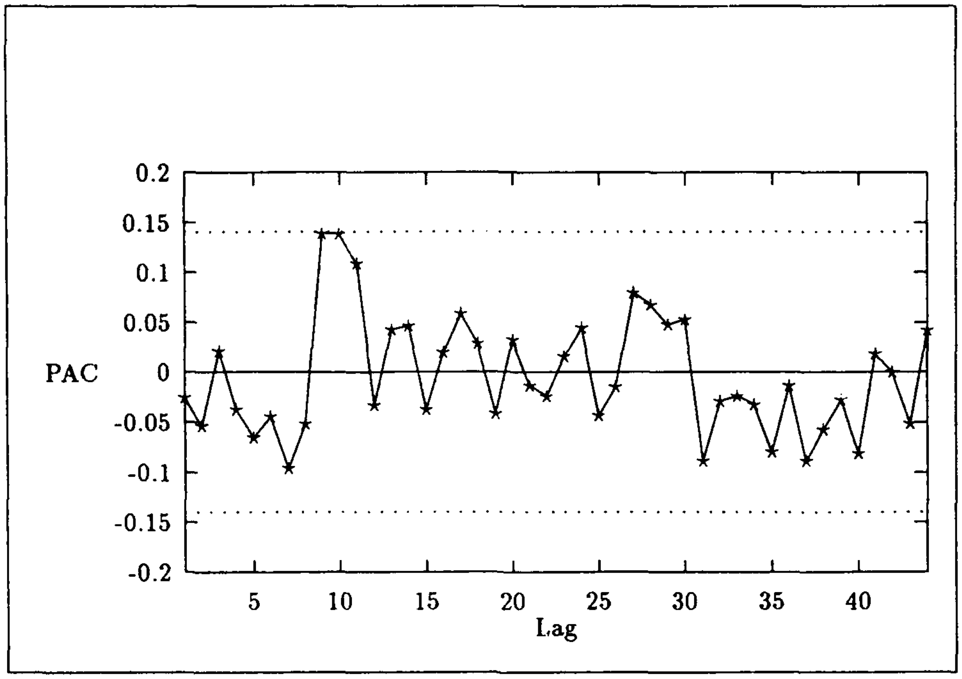


Figure 35. Residual Partial Aut correlations from Fitting the SSTMA$(2_{1,1})_{12}$ Model to the Combined Series of Target Region 11

The SSTMA$(2_{1,1})_{12}$ model can be written as the following equation:

$$x_{2,11,1}(t) = \xi - \sum_{k=1}^{2}\sum_{l=0}^{m_k} \theta_{kl} W_l \epsilon(t-k) - \theta_{sma,l=0} W_0 \epsilon(t-12) + \epsilon(t) \tag{24}$$

Equation 24 for the SSTMA$(2_{1,1})_{12}$ model can be rewritten as:

$$x_{2,11,1}(t) = \xi - \theta_{1,0} W_0 \epsilon(t-1) - \theta_{1,1} W_1 \epsilon(t-1) - \theta_{2,0} W_0 \epsilon(t-2) - \theta_{2,1} W_1 \epsilon(t-2) - \theta_{sma,l=0} W_0 \epsilon(t-12) + \epsilon(t) \tag{25}$$

If the coefficients $\theta_{kl}$ of the parameters are inserted and knowing that $W_0 = I$, Equation 25 can be rewritten as:

$$x_{2,11,1}(t) = 0.27 - 0.424\epsilon(t-1) + 0.160W_1\epsilon(t-1) - 0.396\epsilon(t-2) + 0.131W_1\epsilon(t-2) + 0.586\epsilon(t-12) + \epsilon(t) \tag{26}$$

Figure 36 is a two-dimensional plot of the actual and predicted values of $NHAS_{2j1}$ for target region 11 using the SSTMA$(2_{1,1})_{12}$ model. The values of the predictions for $NHAS_{2,11,1}$ can be found in APPENDIX P.

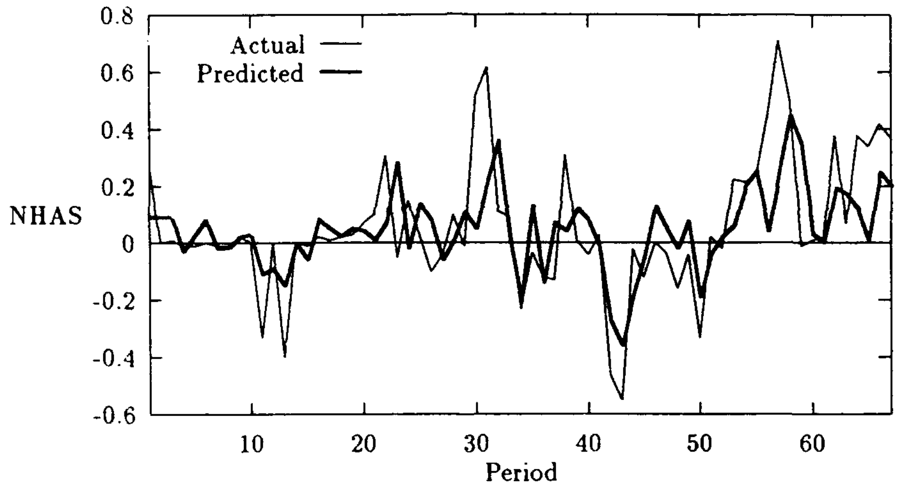


Figure 36. Actual and Predicted Values of $NHAS_{2j1}$ for Target Region 11

Many transformations are required on the fitted and forecasted $NHAS_{2j1}$ values to return to the original historical data. The first transformation that is required converts the fitted $NHAS_{2j1}$ values to fitted $NHA_{2j1}$ values and accounts for the 12 month lag that was taken in an attempt to remove seasonality. The second transformation subtracts the corresponding relative analytical probabilities $\breve{p}_{2j1}$ from the fitted $NHA_{2j1}$ values. This results in $\hat{p}_{2j1}$. It is not possible to forecast $\hat{p}_{2j1}$ into the future without knowing the corresponding analytical $\breve{p}_{2j1}$ value.

Figure 37 is a two-dimensional plot of the actual and predicted values of the relative frequencies of geographical region 11 from January 1985 through July 1991 after all of the transformations have been done to return the forecasting $NHAS_{2j1}$ values for target region 11 back into forecasted values for the historical relative frequencies. The SSR value for the predictions is 2.37. The values of the predictions $\hat{p}_{2,11,1}$ can be found in APPENDIX Q.

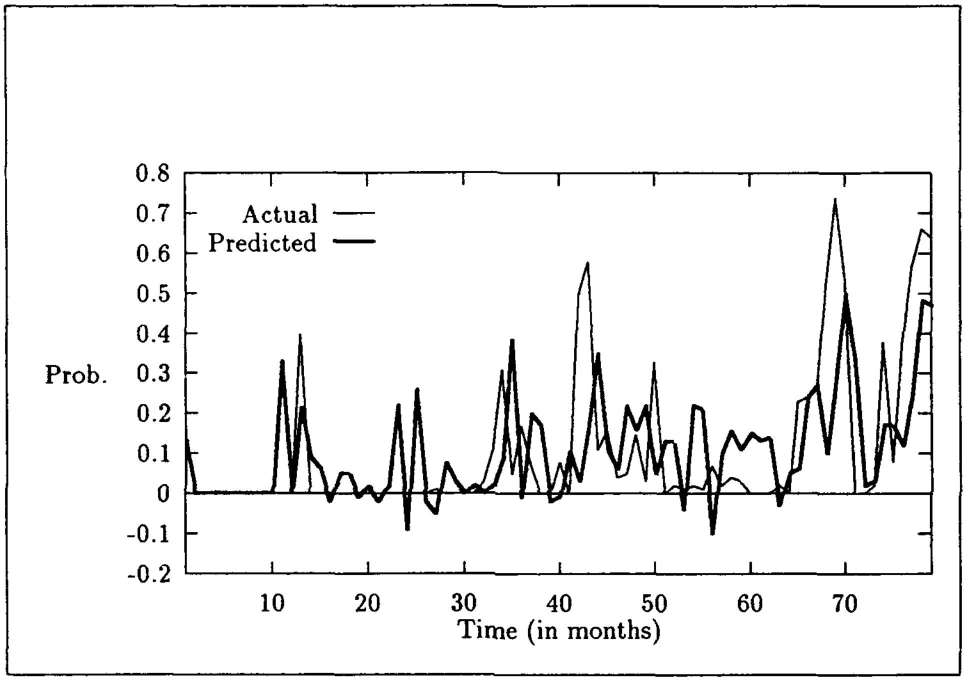


Figure 37. Actual $X_{2j1}$ and Predicted Values $\hat{p}_{2jk}$ of Geographical Region 11

As can be seen in Figure 37, there are a few predictions $\hat{p}_{ijk}$ of geographical region 7 that are negative in value. Since a negative probability is not possible, it is necessary to transform all probabilities with a negative value to probabilities with a zero value. Transforming all the negative probability predictions to zero probability predictions, decreased the SSR value to 2.34. This equates to only a 1.27% decrease in the SSR value, which is not a significant decrease. Figure refAPHIST11T is a two-dimensional plot of the actual and transformed predicted values of geographical region 11. The values of the transformed predictions $\hat{p}_{2,11,1}$ can be fouund in APPENDIX Q.

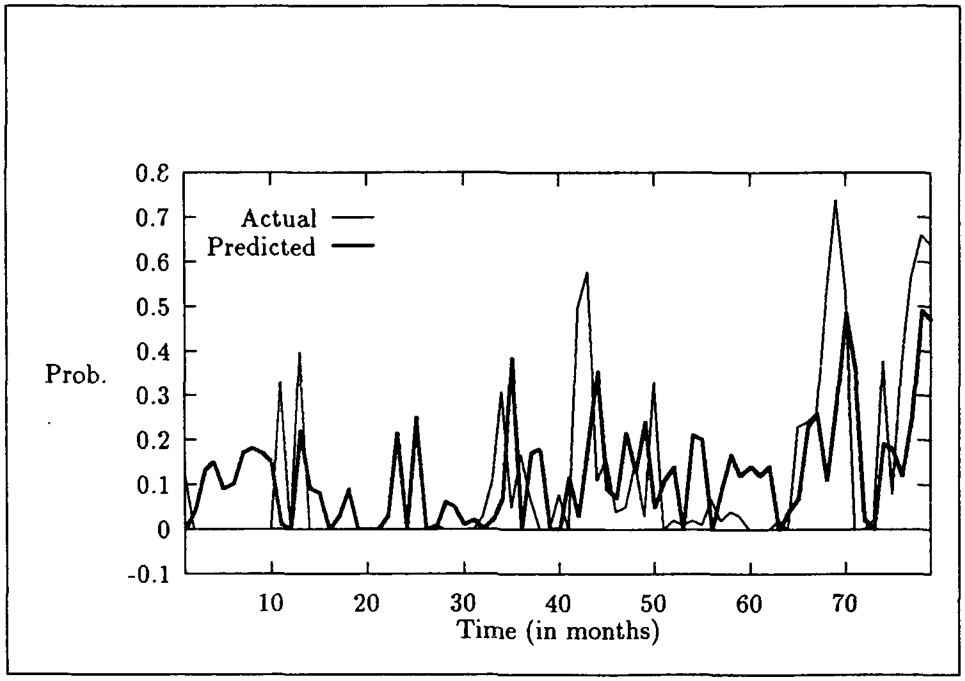


Figure 38. Actual $X_{2j1}$ and Transformed Predicted Values $\hat{p}_{2jk}$ of Geographical Region 7

### 4.16 *Robustness of the Model*

Robustness measures the applicability of the best and most parsimonious model selected for target region 11 to the other geographical regions. To check for the robustness of the target region 11 model, the specified model SSTMA$(2_{1,1})_{12}$ is fit to geographical region 7 . To fit the specified model to geographical region 7, the univariate STARMA model building procedure is followed. The data for target region 7 NHAS$_{2,j,1}$ will be used to fit the predictions and then transformed back into the form of the original data.

Geographical region 7 now become the target region. Since geographical region 7 was selected to be a first order neighbor of target region 11, it is reasonable to select region 11 as a first order neighbor to target region 11. It is reasonable to select the first order neighbors for target region 11 as the first order neighbors for target region 7. Likewise, it is reasonable to select the second order neighbors of target region 11 as the second order neighbors of target region 11. Table 16 lists the first order neighbor regions to target region 7. Table 17 lists the second order neighbor regions to target region 7.

Table 16. First Order Neighbors to Target Region 7

| FIRST ORDER NEIGHBORS |
| --- |
| Region 9 |
| Region 11 |
| Region 13 |
| Region 15 |
| Region 17 |
| Region 19 |

Table 17. Second Order Neighbors to Target Region 7

| SECOND ORDER NEIGHBORS |
| --- |
| Region 1 |
| Region 2 |
| Region 3 |
| Region 4 |
| Region 5 |
| Region 6 |
| Region 8 |
| Region 10 |
| Region 12 |
| Region 14 |
| Region 16 |
| Region 18 |
| Region 20 |
| Region 21 |
| Region 22 |

The only difference between the first order neighbors of target region 7 and the first order neighbors of target region 11 is that region 11 is a first order neighbor of target region 7 and that region 7 is a first order neighbor of target region 11. The second order neighbors of target region 7 are identical to that of target region 11.

The weights for the first order neighbors and second order neighbors of target region 7 were selected in the same fashion of that for target region 7. The first order weights are energy normalized for each season so that the weights sum to 1.0. Because the second order neighbors of target region

7 are identical to that of target region 11, the energy normalized second order weights of target region 11 were used for target region 7. Table 18 lists the weights for region 7 and its first order neighbors for all four seasons. Table 19 lists the weights for region 7 and its second order neighbors for all four seasons.

Table 18. Weights Between Region 7 and its First Order Neighbors

| Region | Winter | Spring | Summer | Fall |
|---|---|---|---|---|
| Region 9 | 0.31 | 0.35 | 0.14 | 0.39 |
| Region 11 | 0.46 | 0.16 | 0.39 | 0.37 |
| Region 13 | 0.20 | 0.13 | 0.16 | 0.09 |
| Region 15 | 0.01 | 0.19 | 0.17 | 0.05 |
| Region 17 | 0.01 | 0.06 | 0.04 | 0.04 |
| Region 19 | 0.01 | 0.11 | 0.10 | 0.06 |

Table 19. Weights Between Region 7 and its Second Order Neighbors

| Region | Winter | Spring | Summer | Fall |
|---|---|---|---|---|
| Region 1 | 0.00 | 0.00 | 0.00 | 0.00 |
| Region 2 | 0.02 | 0.00 | 0.00 | 0.00 |
| Region 3 | 0.03 | 0.02 | 0.00 | 0.04 |
| Region 4 | 0.15 | 0.09 | 0.00 | 0.18 |
| Region 5 | 0.16 | 0.18 | 0.01 | 0.03 |
| Region 6 | 0.26 | 0.09 | 0.08 | 0.15 |
| Region 8 | 0.13 | 0.16 | 0.14 | 0.13 |
| Region 10 | 0.12 | 0.22 | 0.18 | 0.28 |
| Region 12 | 0.08 | 0.11 | 0.13 | 0.15 |
| Region 14 | 0.01 | 0.01 | 0.20 | 0.01 |
| Region 16 | 0.03 | 0.06 | 0.10 | 0.02 |
| Region 18 | 0.01 | 0.06 | 0.10 | 0.02 |
| Region 20 | 0.00 | 0.00 | 0.00 | 0.00 |
| Region 21 | 0.00 | 0.00 | 0.01 | 0.00 |
| Region 22 | 0.00 | 0.00 | 0.00 | 0.00 |

The combined series for target region 7 was developed in the same fashion that the combines series for target region 11 was developed. Figure 39 is a two-dimensional plot of the combined series of target region 7. The values of the combined series with target region 7 can be found in APPENDIX R.

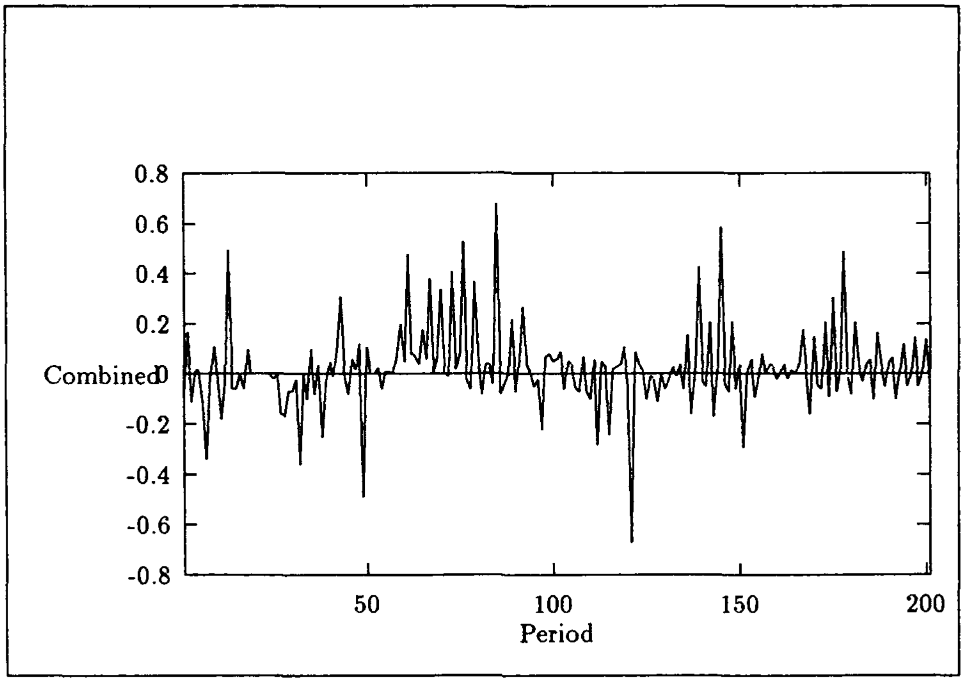


Figure 39. Combined Series of $NHAS_{2,j,1}$ with Target Region 7

The SSTMA$(2_{1,1})_{12}$ model specified using target region 11 was fit to the combined series of target region 7. Figure 40 is a two-dimensional plot of the residual autocorrelations from fitting the specified SSTMA$(2_{1,1})_{12}$ model to the combined series of target region 7. Figure 41 is a two-dimensional plot of the residual partial autocorrelations from fitting the specified SSTMA$(2_{1,1})_{12}$ model to the combined series of target region 7. Figure 42 is a two-dimensional plot of the actual and predicted values of target region 7 using the SSTMA$(2_{1,1})_{12}$ model specified from target region 11. Figure 43 is a two-dimensional plot of the actual and predicted values of the historical data of target region 7 from fitting the specified SSTMA$(2_{1,1})_{12}$ model to the combined series of target region 7. The SSR value for the predicted probabilities is 5.01.

It is obvious from examing Figure 40 and Figure 41, none of the residual autocorrelations or residaul partial autocorrelations were significantly different from zero. The $Q$-statistic for the residual autocorrelations was 27.885. Because the autocorrelations and the partial autocorrelations

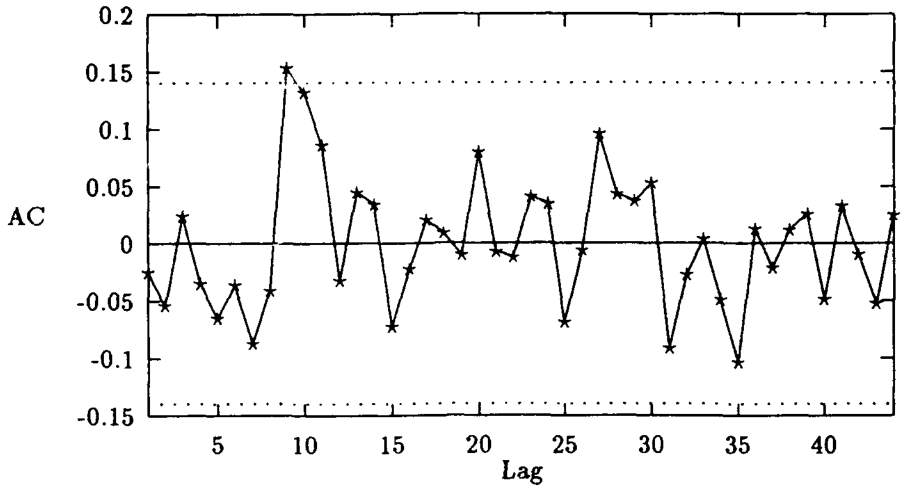


Figure 40. Residual Autocorrelations from Fitting the SSTMA$(2_{1,1})_{12}$ Model to the Combined Series of Target Region 7

are not significantly different from zero, and the $Q$-statistic is smaller than the critical $Q$-value, it is reasonable to assume that the residuals are white noise. The values of the residual autocorrelations and the residual partial autocorrelations from fitting the specified SSTMA$(2_{1,1})_{12}$ model to the combined series of target region 7 can be found in APPENDIX S.

Figure 42 is a two-dimensional plot of the actual and predicted values of target region 7 using the SSTMA$(2_{1,1})_{12}$ model specified from target region 11. Figure 43 is a two-dimensional plot of the actual and predicted values of the historical data of target region 7 from fitting the specified SSTMA$(2_{1,1})_{12}$ model to the combined series of target region 7. The SSR value for the predicted probabilities is 5.01. The values of the fitted $NHAS_{2,7,1}$ can be found in APPENDIX T. The values of the predicted $\hat{p}_{2,7,1}$ can be found in APPENDIX U.

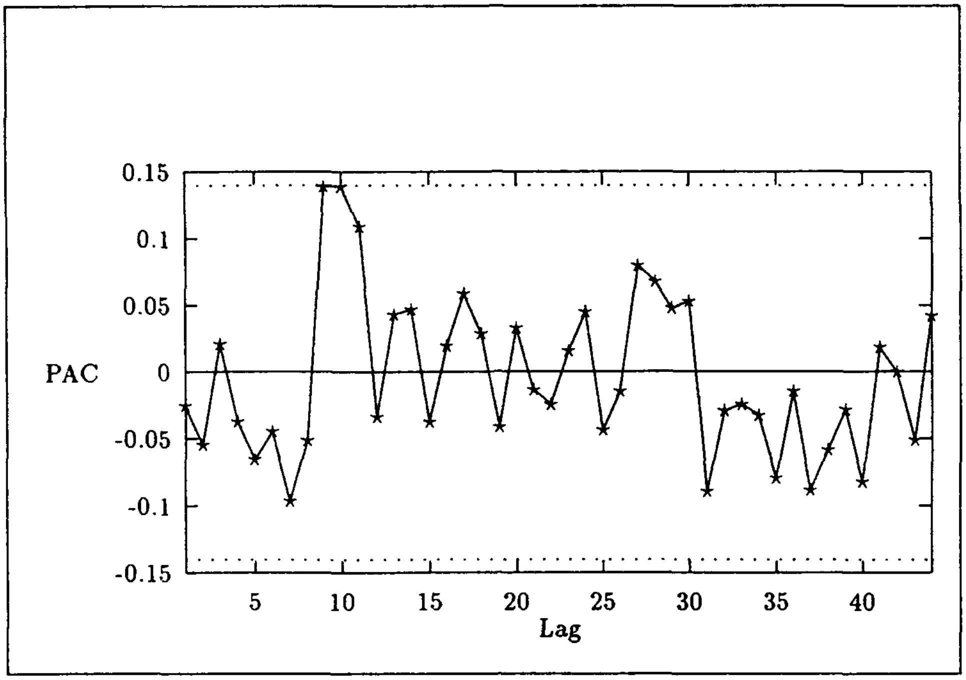


Figure 41. Residual Partial Autocorrelations from Fitting the SSTMA$(2_{1,1})_{12}$ Model to the Combined Series of Target Region 11

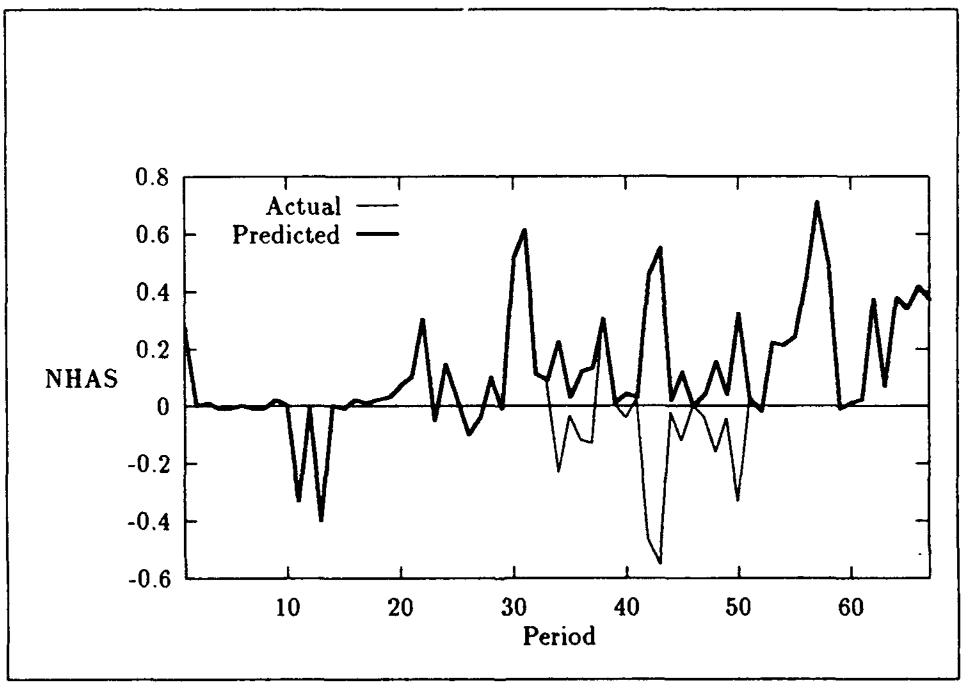


Figure 42. Actual and Predicted Values of $NHAS_{2j1}$ for Target Region 7

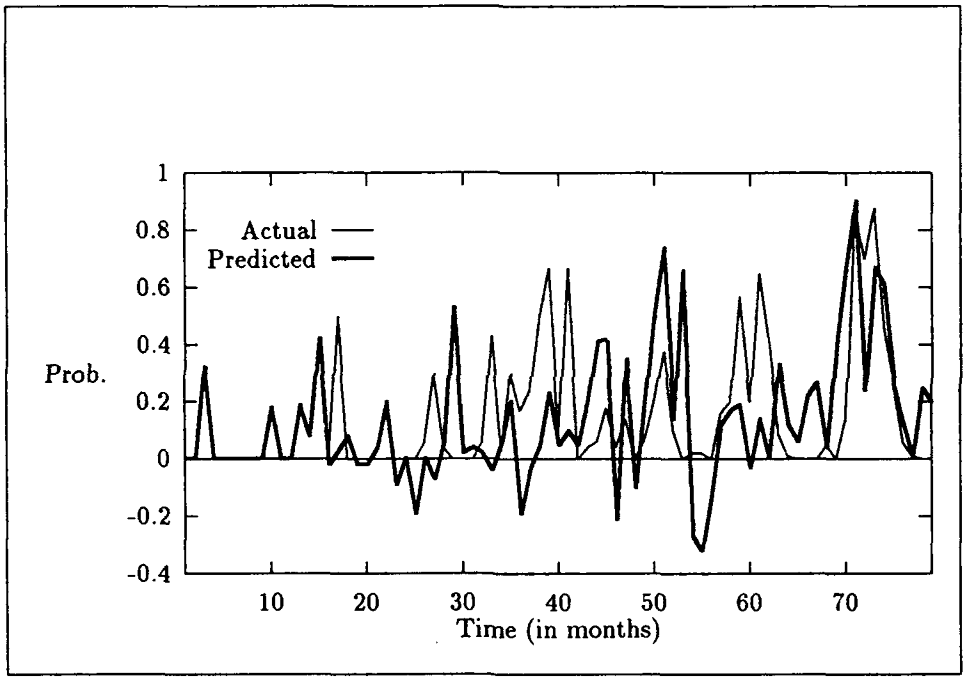


Figure 43. Actual $X_{2j1}$ and Predicted Values $\hat{p}_{2jk}$ of Geographical Region 7

As can be seen from Figure 43, there are several predicted probabilities that are negative in sign. Figure 44 is a two-dimensional plot of the predicted probabilities after all negative predicted probabilities were transformed to zero probabilities. The transformation on the negative predicted probabilities lowered the SSR from 5.01 to 4.47, resulting in a 10.8% decrease in the SSR. The values of the transformed predictions can be found in APPENDIX U.

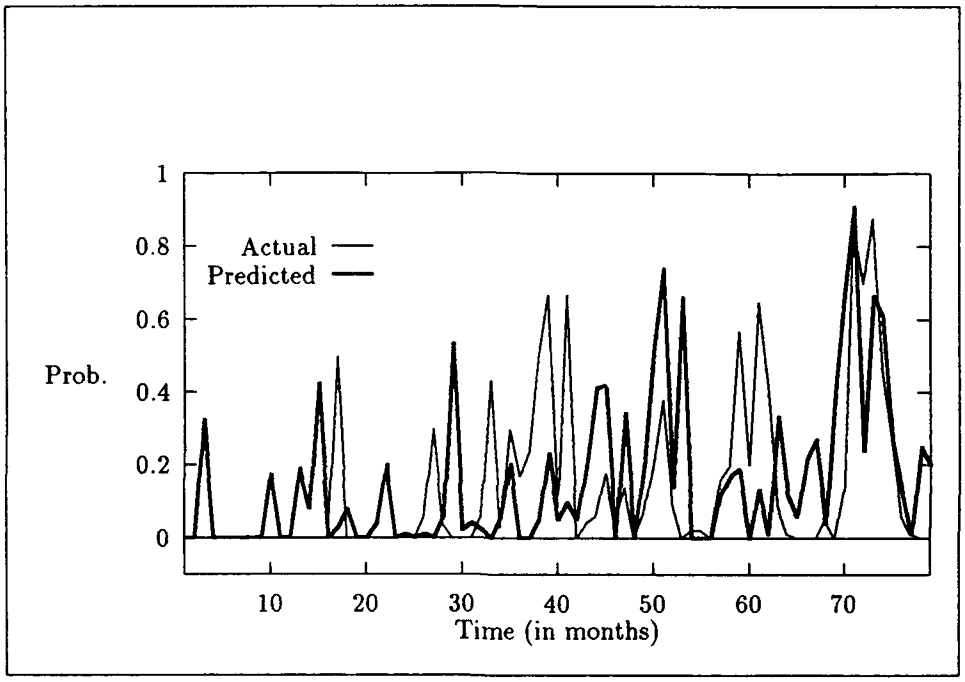


Figure 44. Actual $X_{2j1}$ and Transformed Predicted Values $\hat{p}_{2jk}$ of Geographical Region 7

Next, the an SSTMA$(2_{1,1})_{12}$ model was estimated on the combined series of target region 7.

Table 20 lists the coefficients of the terms in the SSTMA$(2_{1,1})_{12}$ model along with the standard error of the coefficients, the $t$-statistic of the coefficients, and the two tailed significance of the coefficients, all rounded to the nearest one-thousandths.

The two-tailed significance of the coefficient $\theta_{2,0}$ is 0.611, indicating that the coefficient is not significantly different from zero at the 90% confidence level. As a result, it is reasonable to conclude that SSTMA$(2_{1,1})_{12}$ model is not the best and most parsimonious model des-

Table 20. The Identified SSTMA$(2_{1,1})_{12}$ Model Estimated on Target Region 7

| Parameter | Coefficient | Std. Error | T-STAT | 2-Tail Sig. |
|---|---|---|---|---|
| C | 0.023 | 0.010 | 2.423 | 0.015 |
| $\theta_{1,0}$ | 0.297 | 0.064 | 4.613 | 0.000 |
| $\theta_{1,1}$ | -0.147 | 0.064 | -2.281 | 0.023 |
| $\theta_{2,0}$ | 0.033 | 0.065 | 0.509 | 0.611 |
| $\theta_{2,1}$ | 0.267 | 0.064 | 4.160 | 0.000 |
| $sma(12)$ | -0.483 | 0.073 | -6.631 | 0.000 |

ignation for target region 7. The best and most parsimonious model designation for target region 7 may be SSTMA$(2_{1,0})_{12}$. It would not be inappropriate to model target region 7 using a SSTMA$(2_{1,1})_{12}$ model designation. However, it would be inappropriate to model target region 11 using a SSTMA$(2_{1,0})_{12}$ model designation. The $\theta_{2,0}$ coefficient is a significant coefficient in describing and representing target reion 11 and is not a significant coefficient in describing and representing target region 7. Keeping the $\theta_{2,0}$ term in the target region 7 model will not detrimentally effect the predictions since the coefficient is not statistically different from zero. However, removing the $\theta_{2,0}$ term in the target region 11 model will detrimentally effect the predictions. In conclusion, the SSTMA$(2_{1,1})_{12}$ designation may be used to model both target region 7 and target region 11 but the SSTMA$(2_{1,0})_{12}$ designation may not be used to model both target region 7 and target region 11.

The values of the coefficients of the target region 11 estimated SSTMA$(2_{1,1})_{12}$ model and that of the target region 7 estimated SSTMA$(2_{1,1})_{12}$ model were different. Table 21 lists the amount that the parameter coefficients between target region 7 and target region 11 varied.

Because there is such a large percent difference between the coefficients of the estimated target region 7 model and the estimated target region 11 model, it is reasonable to conclude that the estimated SSTMA$(2_{1,1})_{12}$ model for target region 11 should not be used to make predictions for target region 7. However, the SSTMA$(2_{1,1})_{12}$ model designation may be used to predict target region 7.

Table 21. Percent Difference Between the Parameter Coefficients of the Target Region 7 Estimated SSTMA$(2_{1,1})_{12}$ Model and the Target Region 11 Estimated SSTMA$(2_{1,1})_{12}$ Model

| Parameter | Coefficient Difference |
|---|---|
| C | 14.8% |
| $\theta_{1,0}$ | 30.0% |
| $\theta_{1,1}$ | 8.1% |
| $\theta_{2,0}$ | 91.7% |
| $\theta_{2,1}$ | 149.1% |
| $sma(12)$ | 17.6% |

# V. *Intervention Techniques That May Predict $p_{ijk}$*

## 5.1 *Introduction*

A time series of observations can sometimes be effected by external events, commonly referred to as "interventions" (1:355). An example of an intervention is a doctrinal or policy change (7). Most interventions result in either a shift in the mean of the time series or a shift in the trend of the time series (1:355). For purposes of this report, a shift in the mean of the time series will be our focus. This shift in the mean can occur in various forms. Common forms of this shift include step functions where the mean changes abruptly, ramp functions where the mean gradually changes in a linear fashion, and exponential functions where the mean changes in an exponential fashion (7). The form of the shift may even be a combination of different functions (7).

A shift in the mean of the relative probabilities for a given geographical region may be affected by some intervention. It must be remembered however, that the probabilities are relative probabilities and must sum to one. Thus, when the mean of the relative probabilities for a geographical region decreases, the mean relative probability for at least one different geographical region must increase. The opposite is true that when the mean relative probability for a geographical region increases, the mean relative probability for at least one different geographical region must decrease.

Classical statistics require the use of a control group that is not effected by the intervention (3:651). Such a control is not practical to determine a change in a mean value. It is common practice to use a Student's $t$-distribution to test for a change in a mean value (1:355). Unfortunately, the Student's $t$-distribution assumes independent observations and that the change in the mean value can be represented by a step function (1:355). There is usually a correlation, or dependence, among successive data observations in a time series (1:355). Also, a step function may not model the change in the mean (1:355). As a result, the Student's $t$-test is not applicable for intervention analysis in time series data and no quantitative measure of effect of an intervention can be obtained (3:651).

There are several methods that can model interventions to the historical data set to produce estimates of the relative probability of an event type $i$ occurring at geographical region $j$ at time of day $k$. This chapter discusses several techniques that may be appropriate to model interventions and predict $p_{ijk}$. Intervention analysis is the classical method for dealing with external events that effect a time series or a process. There are instances when other methods are preferred. This can occur for several reasons. There may be no experts in the field. The experts may not know the answers to the analyst's questions. The experts may not want to answer the analyst's questions. The analyst may not have the time or the resources to conduct a full and complete intervention analysis. Several techniques other than intervention analysis can be applied to account for interventions: simple exponential smoothing, adaptive response rate exponential smoothing, Kalman filtering, and Multiattirbute Utility Functions. The purpose of this chapter is twofold:

(1) To discuss the methodologies behind each of these intervention methods.

(2) To compare and contrast them.

### *5.2 Intervention Analysis*

When using intervention analysis, a transfer function is generally used to model the effects of an intervention (7). Box and Tiao (1975) use a transfer function-noise model that can describe the effects of an intervention on a time series in the following manner:

$$Y(t) = v(B)I(t) + z(t) \tag{27}$$

where $Y(t)$ is the time series at time $t$,

$v(B)$ is a function that allows for the effect of the intervention,

$I(t)$ is the indicator sequence reflecting the absence and presence of the intervention, and

$z(t)$ is the noise component (1:355).

$I(t)$ is usually a dichotomous variable taking on values of 0 or 1 depending on the presence of the intervention. It is assumed that the noise term $z(t)$ may be modelled by an ARMA process which may be seasonal (3:654). According to Abraham and Ledolter, it is assumed that before the intervention has occurred, the time series can be modelled using the following:

$$E[Y(t)] = z(t) \tag{28}$$

The intervention analysis model in Equation 27 assumes that the time series model parameters are the same before and after the intervention (1:355). The model also assumes that the intervention can be represented as an additive effect of the dichotomous variable $I(t)$ on the noise level (3:654).

Several difficult portions of intervention analysis are:

(1) Determining what external events are interventions to the time series.

(2) Determining if and when an intervention occurs.

(3) Determining exogenously the $v(B)$ function.

As mentioned before, the $v(B)$ function can be a step function, ramp function, exponential functions, or a combination.

A technique called factor analysis can be applied to determine what external events are likely to intervene on the time series. We are interesting in determining what type of intervention is likely to occur that will drastically change the mean value of the time series. A survey given to experts will produce the doctrinal information required.

In order to discuss the methodology of factor analysis, let's suppose that there were nine expert responses to the survey. It is assumed that each expert has approximately the same amount of background and experience. Without this assumption, it would be necessary to weight the results from the survey so that responses from experts with more experience will be weighted more heavily. Let's say that all nine experts said that the subject of interest being modelled never changes due to Event A, Event B, or Event C. Let's suppose there was at least one expert who responded that there is a change in the subject of interest based upon a change in the following:

1. Event D.
2. Event E.
3. Event F.
4. Event G.
5. Event H.

Using the results of the nine surveys, a design matrix $X$ of rank $n$ x $m$ can be created to represent the information where $X_{ij}$ denotes the entry of the $i$ row in the $j$ column for $i = 1, \ldots, n$ and $j = 1, \ldots, m$, where $n$ is the number of expert respondents and $m$ is the total number of potential interventions. In this case, $n = 9$ and $m = 5$. $X_{ij} = 1$ if expert $i$ thinks that a change in $j$ will result in a change in the subject of interest being modelled. The design matrix is:

$$X = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

The dependent $Y$ vector has 9 elements. $Y_i = 1$ if expert$_i$ thinks that changes in the subject of interest occur. The factor analysis has been structured to answer the following question:

> Given an external event $i$ has occurred, what is the probability that the external event will intervene and result in a change in the observations?

To calculate the probability that intervention $i$ (for $i = 1, ..., 5$) can cause a change in the subject of interest, a multiple regression is performed. The serious problem of near singularity will arise if the $Y$ vector contains only values of 1 or only values of 0. Let's suppose than that expert$_6$ stated that changes in the subject of interest never occur. This is represented by $Y_6 = 0$ and $X_{6j} = (\ 0\ 0\ 0\ 0\ 0\ )$. Thus, $Y^T = (1\ 1\ 1\ 1\ 1\ 0\ 1\ 1\ 1)$.

The multiple regression was run on *MicroTSP*. The multiple regression model can be written as the following:

$$Y = .074 + .606z_1 + .170z_2 + .234z_3 + .394z_4 + .053z_5 \tag{29}$$

where $Y = 1$ if a change in the subject of interest occurs,

$Y = 0$ if no change in the subject of interest occurs,

$z_1 = 1$ if a change in EVENT D occurs,

$z_1 = 0$ if no change in EVENT D occurs,

$z_2 = 1$ if a change in EVENT E occurs,

$z_2 = 0$ if no change in EVENT E occurs,

$z_3 = 1$ if a change in EVENT F occurs,

$z_3 = 0$ if no change in EVENT F occurs,

$z_4 = 1$ if a change in EVENT G occurs,

$z_4 = 0$ if no change in EVENT G occurs,

$z_5 = 1$ if a change in EVENT H occurs,and

$z_5 = 0$ if no change in EVENT H occurs.

A coefficient can be interpreted as the probability of the subject of interest changing given that the corresponding event has occurred.

Table 22 shows the coefficient, t-statistic, and 2-tailed significance of each corresponding variable, all rounded to the nearest one-thousandths.

Table 22. Intervention Analysis Multiple Regression Output

| Variable | Coefficient | $t$-stat | 2-Tail Sig. |
|---|---|---|---|
| C | 0.074 | 0.491 | 0.657 |
| $x_1$ | 0.606 | 4.367 | 0.022 |
| $x_2$ | 0.170 | 1.579 | 0.212 |
| $x_3$ | 0.234 | 1.502 | 0.230 |
| $x_4$ | 0.394 | 2.835 | 0.066 |
| $x_5$ | 0.053 | 0.373 | 0.734 |

If we assume an $\alpha$-value of 0.05, it becomes necessary to remove $x_2$, $x_3$, $x_4$, and $x_5$ from the model since these four variables are statistically insignificant, according to the $t$-test. We can conclude that the only external event that can be considered as an intervention is a change in EVENT D. Likewise, if we assume an $\alpha$-value of 0.10, it becomes necessary to remove $x_2$, $x_3$, and $x_5$ from the model. We can conclude that there are two external events that are interventions:

(1) A change in EVENT D.

(2) A change in EVENT G.

If we stay with an $\alpha$-value of 0.05, the model now looks like the following:

$$Y = 0.074 + 0.606z_1 \tag{30}$$

From Equation 30, it can be stated that given a change has occurred in EVENT D, there is a 60.6% chance that the subject of interest will change. This concludes the factor analysis portion of intervention analysis.

The next portion of intervention analysis is determining whether an intervention occurred and if so, when did it occur. There is one recommended way to determine whether and when an intervention occurred in the past that involves two steps:

1. Examining the data plotted over time .
2. ARMA model fitting followed by examination of the residuals and their autocorrelations and partial autocorrelations.

In order to explain the methodologies to evaluate whether and when an intervention occurred, let's suppose we have a univariate time series such that each observation represents the total monthly number of changes in the subject of interest. A possible change in the relative probabilities $p_{ijk}$ can be a doctrinal or policy change that forces events that used to occur in a geographical region to now occur in a different geographical region. Table 23 lists the example subject of interest univariate time series.

It should be noted that the observations of the time series was conjured up to explain intervention analysis. The observations did not come from the historical data base or the analytical data base discussed in earlier chapters.

It is assumed that an external event that has a relatively high probability of effecting the time series has occurred. For purposes of demonstration, the external event will be the start of the Gulf War in January, 1991.

The first step is examination of a plot of the time series. Sometimes, a change in the mean is observable by simply looking at a plot of the time series. Figure 45 is a two-dimensional plot of the monthly number of changes in the subject of interest from August 1989 to July 1992. The $x$-axis

Table 23. Subject of Interest Time Series

| Month and Year | Number of Changes in Observations |
|---|---|
| August 1989 | 25 |
| September 1989 | 22 |
| October 1989 | 24 |
| November 1989 | 27 |
| December 1989 | 21 |
| January 1990 | 24 |
| February 1990 | 22 |
| March 1990 | 26 |
| April 1990 | 27 |
| May 1990 | 31 |
| June 1990 | 25 |
| July 1990 | 28 |
| August 1990 | 27 |
| September 1990 | 25 |
| October 1990 | 23 |
| November 1990 | 24 |
| December 1990 | 24 |
| January 1991 | 27 |
| February 1991 | 28 |
| March 1991 | 42 |
| April 1991 | 57 |
| May 1991 | 59 |
| June 1991 | 68 |
| July 1991 | 71 |
| August 1991 | 97 |
| September 1991 | 102 |
| October 1991 | 93 |
| November 1991 | 99 |
| December 1991 | 100 |
| January 1992 | 99 |
| February 1992 | 99 |
| March 1992 | 100 |
| April 1992 | 100 |
| May 1992 | 101 |
| June 1992 | 100 |
| July 1992 | 99 |

is set up such that $x = 1$ corresponds to August 1989, $x = 2$ corresponds to September 1989...., and $x = 36$ corresponds to July 1992.

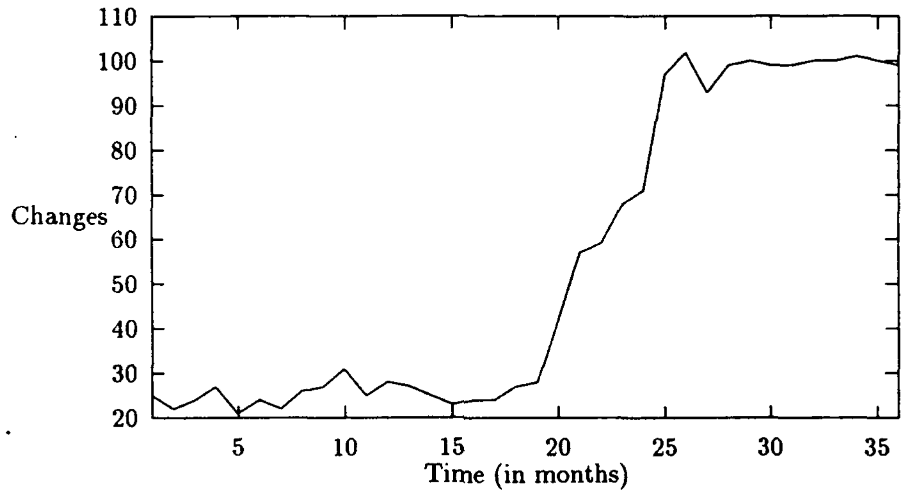


Figure 45. Monthly Number of Changes in the Subject of Interest

The monthly number of the subject of interest changes appears to fluctuate around 25 before the Gulf War began in January 1991. After January 1991, there appears to be an increase in the monthly number of the subject of interest changes and then a levelling off around July 1991 or August 1991. After August 1991, the monthly number of changes in the subject of interest appear to fluctuate around 100.0. The mean of the monthly number of changes in the subject of interest before January 1991 is 25.0. After January 1991, the mean is 84.111. From an examination of the time series plotted over time and the means of the number of monthly subject of interest changes over time, there appears to be an effect on the time series.

The next step is accomplished by fitting the time series with an ARMA model (3:655). It should be noted that the estimates created by this model are the $z(t)$ terms for the Box-Jenkins transfer function model specified earlier in Equation 27. An ARMA model is fit using observations dating back before the intervention presumably occurred. Using *MicroTSP*, the best and most parsimonious ARMA model is an AR(1) model with a constant value of 24.984 and an AR(1) coefficient of 0.202. The analyst fits the specified AR(1) model to the entire time series. If the residuals exhibit no special trend or characteristic and there is no significant increase in their standard error, then no significant intervention has occurred. Figure 46 is a two-dimensional plot of the actual and predicted values of the subject of interest using an AR(1) model that was specified using the observations that occurred before the start of the Gulf War. The $x$-axis is set up such that $x = 1$ corresponds to August 1989, $x = 2$ corresponds to September 1989,..., and $x = 36$ corresponds to July 1992.

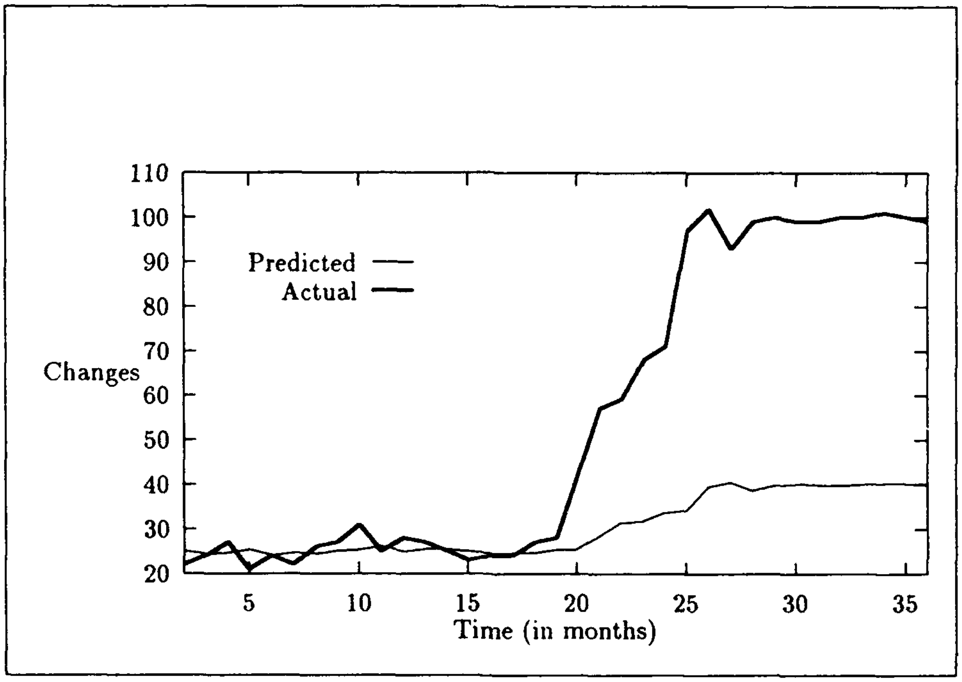


Figure 46. Actual and Predicted Values of the Subject of Interest Using an AR(1) Model

Figure 47 is a two-dimensional plot of the residuals from fitting the subject of interest with an AR(1) model. The $x$-axis is set up such that $x = 1$ corresponds to August 1989, $x = 2$ corresponds to September 1989,..., and $x = 36$ corresponds to July 1992. The plot of the residuals shows that the AR(1) model does not fit the time series after January 1991.

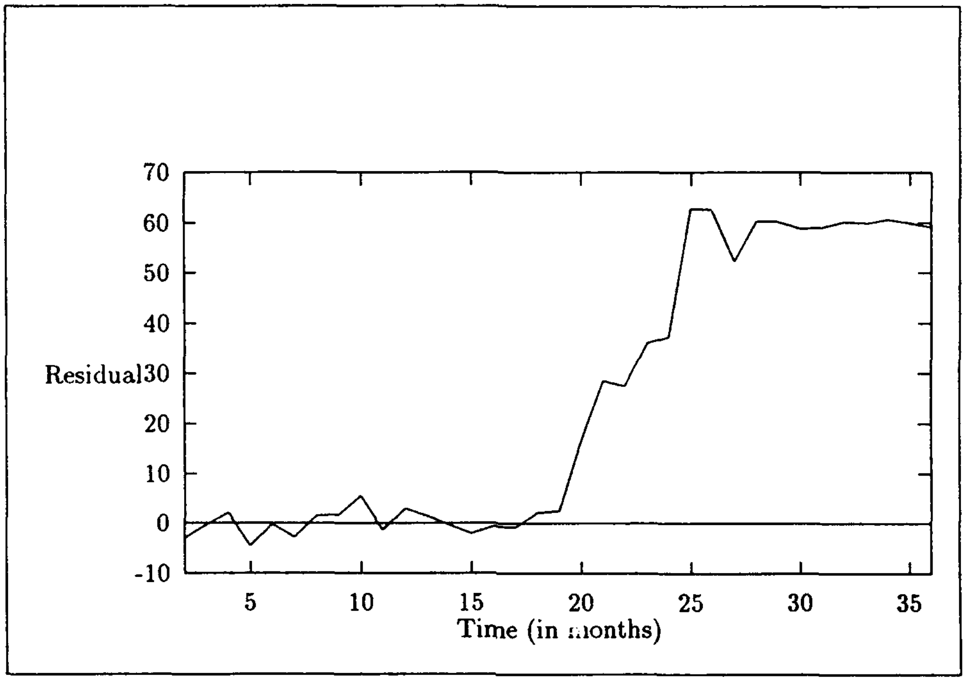


Figure 47. Residuals from Fitting the Subject of Interest with an AR(1) Model

Figure 48 is a two-dimensional plot of the autocorrelations of the residuals from fitting the subject of interest with an AR(1) model. It is obvious from Figure 48 that after January 1991, all of the residuals are significantly different from zero. The autocorrelations follow a definite pattern. The autocorrelations for the first 11 lags are all positive and the autocorrelations for lags 13 through 20 are all negative.

Figure 49 is a two-dimensional plot of the partial autocorrelations of the residuals from fitting the subject of interest with an AR(1) model.

The analyst now determines the best and most parsimonious ARMA model for the entire time series as an AR(1) model with a constant value of 290.306 and an AR(1) coefficient of 0.995.

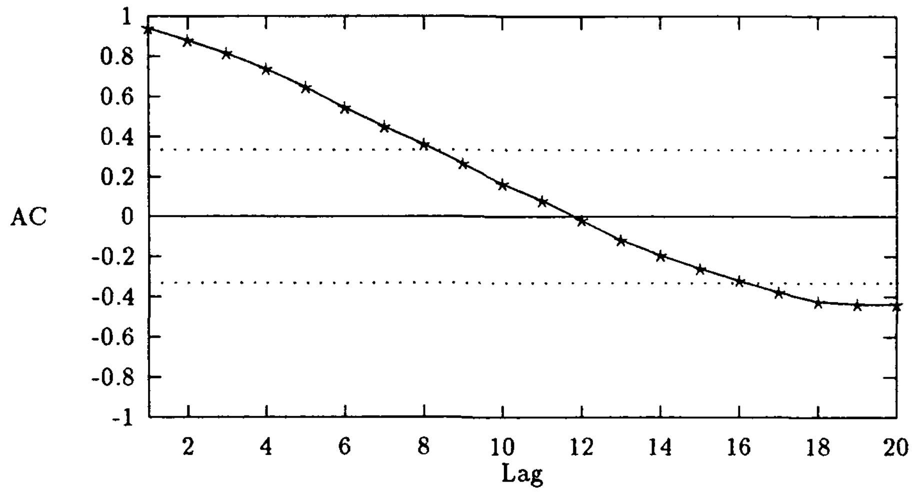


Figure 48. Autocorrelations of Residuals from Fitting the Subject of Interest with an AR(1) Model

It can be concluded that the Gulf War is an intervention on the monthly number of changes in the subject of interest. The following led to the conclusion that the Gulf War is an intervention:

1. The plot of the time series.
2. The differences in coefficients of the AR(1) model that was determined from the data dating back before the Gulf War and the AR(1) model determined from all of the data.
3. The plot of the residuals of the time series fitted with the AR(1) model determined from the data before the Gulf War.
4. The plots of the residual autocorrelations and the residual partial autocorrelations.

The next portion of intervention analysis is estimating the $v(B)$ function of all interventions. The $v(B)$ function must be calibrated exogenously from the data. One of the ways to calibrate the

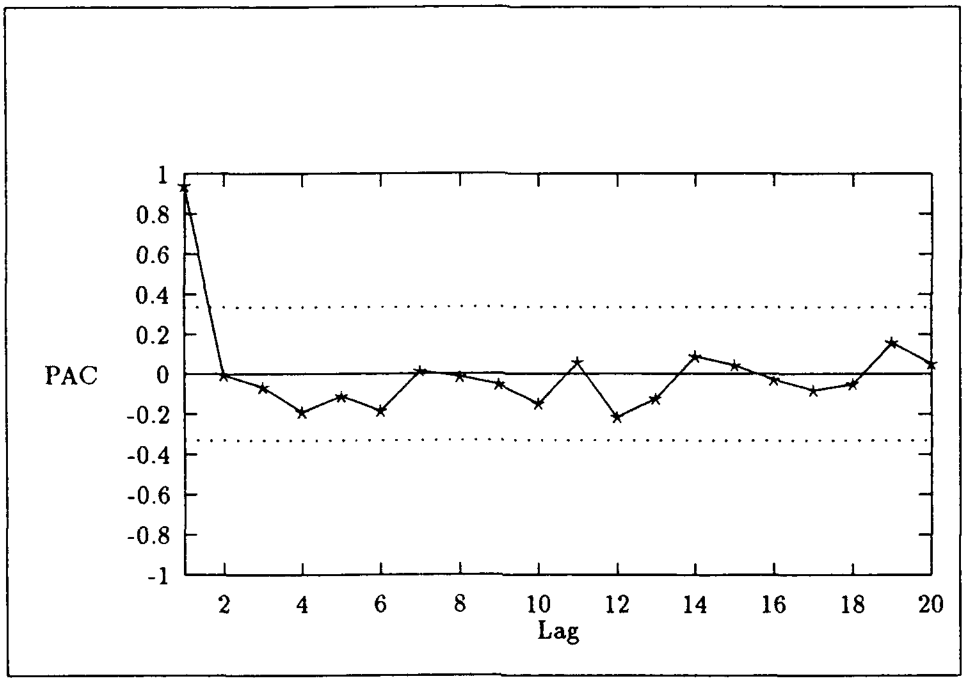


Figure 49. Partial Autocorrelations of Residuals from Fitting the Subject of Interest with an AR(1) Model

$v(B)$ function is to apply the fractile method which involves questioning a group of experts on the form and shape of the $v(B)$ function. The fractile method is commonly used to measure utility and the $v(B)$ function can be modelled as a utility function. Here is an example of a generalized $v(B)$ questionnaire.

### *5.3 Generalized Fractile Method Survey*

*5.3.1 Note.* This survey is a generalized survey using the fractile method. The analyst must fill in all areas of this generalized survey in parenthesis (). For example, everywhere there is (INTERVENTION), the analyst must fill in the specific intervention that is being mentioned.

*5.3.2 **Purpose of this Study**.* In this section, it is important that the analyst states why the survey is being conducted and who the survey is being conducted for. The fractile approach should be mentioned and briefly described.

*5.3.3 How This Survey Will Be Used.* In this section, the analyst should describe how the responses from this survey will be used. The calibration of the $v(B)$ function using the results should be mentioned. It may also prove useful to state why the $v(B)$ function must be calibrated.

*5.3.4 Instructions Regarding This Survey.* In this section, the analyst should state something like the following: Answer all questions to the best of your ability. If you are not sure about an answer or can not answer due for any given reason, state so (i. e., "To the best of my knowledge", "I don't know", et cetera).

If you have any questions regarding this survey, my name is (NAME) and I can be reached at (PHONE NUMBER). My office is (OFFICE).

Your Name and Rank:

Your Office:

Experience:

It has been determined that (INTERVENTION) is an external event that has an effect on (SUBJECT OF INTEREST). When is (INTERVENTION) likely to happen?

When has (INTERVENTION) occurred in the past?

When (INTERVENTION) occurs, what type of effect does it have on (SUBJECT OF INTEREST)? In other words, will the effect resemble a ramp function, step function, exponential function, or a combination of functions?

When (INTERVENTION) occurs, will the mean value of (SUBJECT OF INTEREST) increase, decrease, or both?

When (INTERVENTION) occurs, what is the most likely value of (SUBJECT OF INTEREST)?

The last time (INTERVENTION) occurred, what was the value of (SUBJECT OF INTEREST)?

After (INTERVENTION) occurs, what is the (MAXIMUM / MINIMUM) value that (SUBJECT OF INTEREST) will be?

What is the absolute difference between the (MAXIMUM / MINIMUM) value of (SUBJECT OF INTEREST) and the value of (SUBJECT OF INTEREST) when it last occurred? If (INTERVENTION) has never occurred, substitute "most likely value of (INTERVENTION)" for "value of (INTERVENTION) when it last occurred."

What is that absolute value divided by $N$? ($N$ is determined by the analyst and is equal to 1 plus the number of intermediate values to be used in the calibration of $v(B)$)

When is the (SUBJECT OF INTEREST) most likely to (INCREASE / DECREASE) to (INTERMEDIATE VALUE)?

NOTE: The (INTERMEDIATE VALUE)s can be calculated by multiplying the value of the absolute value divided by $N$ with values between 2 through $N-1$.

Repeat the last question for all intermediate values.

When is the (MAXIMUM SUBJECT OF CHANGE) most likely to occur?

Thank you for your time and effort.

### *5.4 Fractile Method*

Since we assumed an $\alpha$-value of 0.05, the only intervention that is statistically significant is a change between EVENT D. Let's assume that we are interested in calibrating $v(B)$ as the total number of changes in observations as the Gulf War broke out in January 1991.

The first step in calibrating the $v(B)$ function is to determine what form of $v(B)$ is most likely to occur starting in January of 1991 by examining a plot of the time series of interest (7).

From examination of the time series, let's assume that the analyst believes that the form of $v(B)$ is either an exponentially increasing function or a ramp function.

On average, 25 changes in the subject of interest were made before January. The analyst asks the experts what is the maximum monthly number of changes in the observations that will occur after the Gulf War started. The experts will probably disagree on the exact number, but the analyst should not allow the experts to discuss it. Rather, the analyst should weight the response of each individual expert as a function of the expert's background and knowledge of changes in the subject of interest. For example, an analyst would not want to place a high weight on the responses of $\text{expert}_6$ because that expert erroneously stated that no changes ever occur in the observations. The same nine experts that helped the analyst perform the factorial analysis are used, but this time, it is assumed that the experts differ in their background and knowledge of the subject of interest. Table 24 lists the rankings of the experts, where 1 is the lowest rank and 9 is the highest rank.

Table 24. Rankings of Nine Experts

| Expert | Rank |
|---|---|
| 1 | 3 |
| 2 | 7 |
| 3 | 2 |
| 4 | 6 |
| 5 | 8 |
| 6 | 1 |
| 7 | 4 |
| 8 | 9 |
| 9 | 5 |

A simple way to apply the ranks is the following:

1. Multiply an expert's response by his ranking.
2. Add up all the weighted responses.
3. Divide the weighted response by the sum of the ranks (1, 2, ..., 9).

Table 25 shows the experts' responses for the maximum monthly number of changes in the subject of interest that will occur after the Gulf War started.

Table 25. Experts' Responses to Maximum Monthly Number of Changes

| Expert | Response | Rank | Weighted Response |
|---|---|---|---|
| 1 | 90 | 3 | 270 |
| 2 | 120 | 7 | 840 |
| 3 | 110 | 2 | 220 |
| 4 | 100 | 6 | 600 |
| 5 | 85 | 8 | 680 |
| 6 | 130 | 1 | 130 |
| 7 | 70 | 4 | 280 |
| 8 | 100 | 9 | 900 |
| 9 | 120 | 5 | 600 |

Sum of the Weighted Responses = 4500.0

Sum of the Ranks = 45

Weighted Sum = $\frac{4500}{45} = 100$

The weighted average response for the maximum number of monthly changes is 100. The next step is to ask the experts in what month will the total number of changes in the subject of interest increase to some intermediate value between 25 and 100. Let's use 50. Table 26 list the responses of the nine experts.

Table 26. Experts' Responses to When 50 Changes Will Occur

| Expert | Response | Rank | Weighted Response |
|---|---|---|---|
| 1 | July (6) | 3 | 18 |
| 2 | February (1) | 7 | 7 |
| 3 | May (4) | 2 | 8 |
| 4 | April (3) | 6 | 18 |
| 5 | May (4) | 8 | 32 |
| 6 | June (5) | 1 | 5 |
| 7 | April 15 (3.5) | 4 | 14 |
| 8 | May 15 (4.5) | 9 | 40.5 |
| 9 | August 15 (7.5) | 5 | 37.5 |

Note: The number in parenthesis ( ) is the number of months that response is from January. For example, April is 3 months away from January.

Sum of the Weighted Responses = 180

Sum of the Ranks = 45

Weighted Sum = $\frac{180}{45} = 4 \Rightarrow$ MAY

The calculated average weighted response for an increase to 50 monthly changes is May. Following the same line of questioning, ask the experts in what month will the total number of changes increase to another intermediate value, such as 75. It is the analyst's responsibility to ensure that the responses from the experts are reasonable. For example, assuming that the function of the $v(B)$ function is increasing, if an expert stated that the number of monthly changes would increase from to 50 in March and then states that the number of monthly changes would increase to 75 in February of the same year, the expert is being unreasonable. Table 27 shows the responses for when the total monthly number of changes will increase to 75.

Table 27. Experts' Responses to When 75 Changes Will Occur

| Expert | Response | Rank | Weighted Response |
|---|---|---|---|
| 1 | Sept (8) | 3 | 24 |
| 2 | April (3) | 7 | 21 |
| 3 | July (6) | 2 | 12 |
| 4 | June (5) | 6 | 30 |
| 5 | August (7) | 8 | 56 |
| 6 | August (7) | 1 | 7 |
| 7 | June (5) | 4 | 20 |
| 8 | July (6) | 9 | 54 |
| 9 | Sept (8) | 5 | 40 |

Note: The number in parenthesis ( ) is the number of months that response is from January.

Sum of the Weighted Responses = 264

Sum of the Ranks = 45

Weighted Sum = $\frac{264}{45} = 5.8667 \Rightarrow$ JULY

Rounding to the nearest digit (month), the calculated average weighted response for an increase to 75 monthly changes is July.

The analyst may continue the line of questioning for intermediate values. The final question for the experts is when will the number of monthly changes increase to the maximum number, already determined to be 100. Table 28 summarizes the experts' responses.

Table 28. Experts' Responses to When Maximum Changes Will Occur

| Expert | Response | Rank | Weighted Response |
|---|---|---|---|
| 1 | Oct (9) | 3 | 27 |
| 2 | June (5) | 7 | 35 |
| 3 | Sept (8) | 2 | 168 |
| 4 | August (7) | 6 | 42 |
| 5 | Sept (8) | 8 | 64 |
| 6 | Sept (8) | 1 | 8 |
| 7 | July (6) | 4 | 24 |
| 8 | August 15 (7.5) | 9 | 67.5 |
| 9 | Oct (9) | 5 | 45 |

Note: The number in parenthesis ( ) is the number of months that response is from January.

Sum of the Weighted Responses = 328.5

Sum of the Ranks = 45

Weighted Sum $= \frac{328.5}{45} = 7.3 \Rightarrow$ AUGUST

Rounding to the nearest month, the calculated average weighted response for an increase to 100 changes is August.

After the questioning is complete, the analyst will have a linear piecewise approximation to the $v(B)$ function. The analyst initially thought that the $v(B)$ function would be either an exponentially increasing function or a ramp function. Figure 50 is a plot of the actual monthly number of changes in observations, a fit to the data using a constant slope of 10.714 from January 1991 to July 1991, and a linear piecewise approximation to the $v(B)$ function.

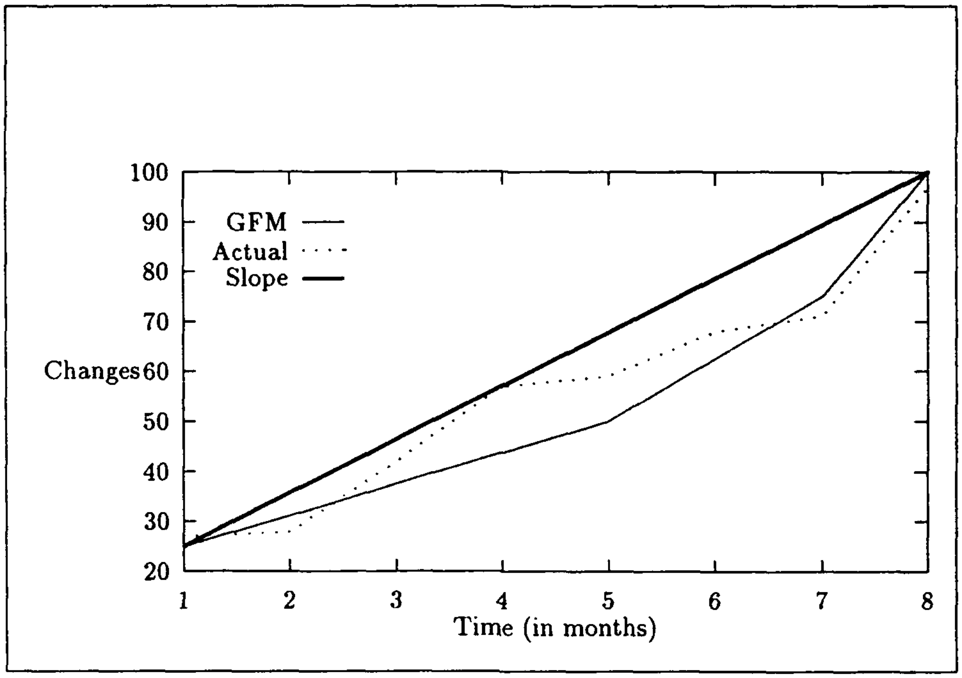


Figure 50. Actual and Predicted Values using the Generalized Fractile Method and a Constant Slope for the Subject of Interest

If the actual data is fitted using a constant slope of 10.714, the mean squared error from January 1991 to July 1991 is 571.0. If the actual data is fitted using the linear piecewise approximation to the $v(B)$ function, the mean squared error from January 1991 to July 1991 is 337.0. The linear piecewise approximation resembles an exponentially increasing function more than a ramp function and the mean squared error is considerably lower for the piecewise linear approximation. Thus, it can be concluded that the ramp function is not appropriate for the calibration of the $v(B)$ function.

### 5.5 *Simple Exponential Smoothing*

Simple exponential smoothing is a popular technique to forecast time series due to its simplicity, ease of computation, accuracy, and responsiveness to changes in the data. A forecasting system utilizing simple exponential smoothing involves redesignating the model parameters each period in order to incorporate the most recent period's observation (16:82). An appropriate way to obtain the new estimate is to modify the old estimate by some fraction of the forecast error resulting from using the old estimate to forecast demand in the current period from the following:

$$F(t+1) = \alpha X(t) + (1-\alpha)F(t) \tag{31}$$

where $F(t)$ is the forecast at time $t$,

$X(t)$ is the observation at time $t$, and

$\alpha$ is called the smoothing constant (16:84) (25:53).

Typical values for $\alpha$ lie between 0.01 and 0.30. The $\alpha$-value is selected by minimizing the mean squared error. Any system that has an $\alpha$-value greater than 0.50 should not be modelled using simple exponential smoothing.

A relatively small $\alpha$ is used when lots of noise is present in the data. A small $\alpha$ (0.01 - 0.10) will force the forecasting system to react very slowly to changes in the data. As a result, the system will smooth over the noise. A relatively large $\alpha$ (0.20 - 0.30) is used when it is necessary to react quickly to changes in the data.

A problem arises when an intervention occurs that abruptly effects the underlying process. Values of $\alpha$ between 0.01 and 0.30 may take a long time to home in to the new level; biased forecasts will occur and will continue for some time (25:53).

An intervention, such as the start of the Gulf War, effects the time series abruptly. A relatively high value of $\alpha$ is expected to model the effect of an intervention. Let's assume that the analyst is interested in simple exponential smoothing the monthly number of changes in the subject of interest. Using the software package *MDECAST*, simple exponential smoothing with an $\alpha$-value of 1.0 minimized the mean square error. A simple exponential smoothing system that minimizes the mean square error when $\alpha = 1.0$, is simply using the most recent observation as the prediction one time period into the future. Table 29 lists the $\alpha$-value and the associated mean squared value , rounded to the nearest one-thousandths, for the time series of number monthly changes from August 1989 through August 1992.

Table 29. Simple Exponential Smoothing Results

| ALPHA | MSE |
|---|---|
| 0.10 | 597.472 |
| 0.20 | 265.864 |
| 0.30 | 155.023 |
| 0.40 | 104.789 |
| 0.50 | 77.952 |
| 0.60 | 62.227 |
| 0.70 | 52.490 |
| 0.80 | 46.280 |
| 0.90 | 42.311 |
| 0.99 | 40.074 |
| 1.00 | 39.889 |

The simple exponential smoothing approach is a very simple and elementary way to account for interventions. Though the computer runs show that a very high $\alpha$ value produces the best mean squared error, it is not wise to use an $\alpha$ value higher than 0.50. This is not wise because a system with an $\alpha$ of 0.50 will not respond quickly to interventions, and it will also respond to noise. There is a tradeoff between modelling abrupt changes in the data due to an intervention and modelling noise. A multiple criteria decision making approach of goal setting programming and compromise programming to determine the $\alpha$ value will produce better results than the $\alpha$ value that minimizes the mean squared error. According to Chan,

> Goal setting programming for satisficing solutions is defined as the procedure of identifying a satisficing set $S$ such that, whenever the decision outcome is an element of $S$, the decision maker will be happy and satisfied and is assumed to have reached the optimal solution. (6:1)

No system will ever completely model the interventions effecting the time series, while completely smoothing the noise. For purposes of illustration, let's assume that the decision maker (DM) is the one person that will determine if the model is appropriate. The DM defines his satisficing set as $S = (Y_1, Y_2 \mid Y_1 \geq 0.90, Y_2 \geq 0.90)$ where $Y_1$ is the percentage of the noise that is smoothed and $Y_2$ is the percentage of the intervention(s) that is modelled.

The satisficing set is not reachable using a simple exponential smoothing approach. The DM is not willing to lower his standards and so it becomes necessary to employ compromise programming. The compromise solution is that solution that minimizes the norm. The DM decides that it is equally important to model the intervention(s) and smooth the noise.

There is a direct relationship between the half-life of an intervention and the best $\alpha$ value. The half-life of an intervention is considered to be that point in time where the change in the mean has reached its half-way point. For example, if the number of observations changes was 25 in January and 100 in August, the half-life is that month where the number of changes was approximately 37.5. In the example, this corresponds to the month of April. This was 3 months after the intervention occurred. If the change in the mean changes very abruptly, a relatively high value of alpha is necessary to stay "caught up" with the effect of the intervention. If the change in the mean occurs slowly, a relatively small value of $\alpha$ will fit the intervention.

There is also a direct relationship between the effect of the intervention and the $\alpha$ value. If the percent change in the mean is relatively large, a relatively large $\alpha$ value is necessary to accommodate the large change. If the percent change in the mean is relatively small, a relatively small $\alpha$ value will suffice.

The amount of noise in the data can be determined from a plot of the autocorrelations of the time series and the $Q$-statistic. An autocorrelation statistically different from zero signifies non-randomness. Higher values of the $Q$-statistic suggest more noise in the data. The capability to model a system decreases as the $Q$-statistic increases. Thus, the $Q$-statistic effects the percentage of noise smoothed and the percentage of the intervention(s) modelled.

Let's define the $X$-space as three-dimensional with $x_1$, $x_2$, and $x_3$ where $x_1$ is the half-life of the intervention, $x_2$ is the percent change in the mean, and $x_3$ is the $Q$-statistic. The following relationships are assumed to exist:

$$Y_1(\%\ \textit{intervention modelled}) = f(x_1, x_2, x_3) \tag{32}$$

$$Y_2(\%\ \textit{noise smoothed}) = f(x_1, x_2, x_3) \tag{33}$$

$$Z(\textit{best}\ \alpha - \textit{value}) = f(Y_1, Y_2) \tag{34}$$

It is assumed that the above relationships exist such that the $X$-, $Y$-, and $Z$- spaces are convex and continuous. For purposes of illustration, let's assume the following:

$$G_1 = \frac{x_1}{100} + x_2 \leq 1.20 \tag{35}$$

$$G_2 = x_3 \leq 43 \tag{36}$$

$$Y_1 = \frac{0.50x_1 + 35x_2 + 0.15x_3}{100} \tag{37}$$

$$Y_2 = \frac{0.60x_3}{100} \tag{38}$$

$$Z = Y_1 + Y_2 \tag{39}$$

The compromise solution can be written as:

$$MIN[0.90 - \frac{0.50x_1 + 0.35x_2 + 0.25x_3}{100} + 0.90 - \frac{-0.60x_3}{100}] \tag{40}$$

S.T. $G_1 \leq 1.20$

$G_2 \leq 43$

This can be solved on any linear programming solver.

### *5.6 Adaptive Response Rate Exponential Smoothing*

Simple exponential smoothing assumes that the process is constant (16:81). This assumption requires a constant mean and variance. Differencing is a common cure for varying means and a transformation is a common cure for varying variance.

If a time series contains a varying mean or variance, the forecasts produced by exponential smoothing are not statistically meaningful. Classical statistical theory treats the majority of data changes as random effects or temporary shifts. Examples of such data changes include steps, trends, and transient shifts. If these changes are permanent, it is likely that a new forecasting model will have to be specified to deal with the new equilibrium conditions. There are two potential problems with the classical statistical forecasting methods:

(1) They may not detect a shift in the data .

(2) They may not model well after a shift has occurred since the model is using the same specifications from before the shift.

If there is a change in the mean, adaptive response rate exponential smoothing can account for the change in the mean by continuously updating its parameter. Updating the parameters accounts for changes in the pattern and can deal with changes in trend.

Once an $\alpha$ level for a simple exponential smoothing system has been specified, it is not usually changed until a new model is created. When using a simple exponential smoothing model, a tracking signal is normally used to aid in the detection of an inappropriate $\alpha$-value. It is sometimes difficult to detect a "bad" tracking signal and realize an intervention has occurred when forecasts are being made for several thousand time series (25:54).

The tracking signal is defined as:

$$TrackingSignal = \frac{smoothed\ error}{\mid smoothed\ error \mid} \tag{41}$$

If the model is appropriate, the tracking system will fluctuate around zero. The most obvious way to react automatically when fitted values go out of control is to increase the value of $\alpha$, so as to give more weight to recent data in order to model the changes in the data (25:54-55). It is important to lower the $\alpha$-value once the changes have occurred in order to reduce the amount of noise modelled (25:55). A very elementary way of adapting the response rate is to equate the $\alpha$-value to the modulus of the tracking signal (25:55). The equation for adaptive response rate exponential smoothing is:

$$F(t+1) = \alpha(t)X(t) + [1 - \alpha(t)]F(t) \tag{42}$$

such that

$$\alpha(t+1) = \mid \frac{e(t)}{M(t)} \mid \tag{43}$$

$$E(t=1) = M(t=1) = 0 \tag{44}$$

$$E(t) = [1 - \alpha(t)]e(t) + \alpha(t)E(t-1) \tag{45}$$

$$M(t) = [1 - \alpha(t)] \mid e(t) \mid + \alpha(t)M(t-1) \tag{46}$$

An adaptive response rate exponential smoothing forecast was created using the software package *MDECAST*. The mean squared error using this approach was 54.515. A mean squared error of 52.490 occurred when $\alpha$ was 0.70 for the simple exponential smoothing. A simple exponential smoothing system with an $\alpha$-value of 0.70 is considered an inappropriate model since the $\alpha$-value is greater than 0.50. Thus, it is more advantageous to use an adaptive response rate exponential smoothing forecast.

### *5.7 Kalman Filter*

The adaptive response rate exponential smoothing can account for change in the mean of the data. Unfortunately, it can not account for changes in the variance of the data. If intervention analysis can not be accomplished, the Kalman filter is the best way to account for the effects of an intervention because it can account for variable models, variable parameters, and variable variances simultaneously. All forecasting methods are special cases of the Kalman filter.

The Kalman filter combines two independent estimates of the time series to form a weighted estimate. One estimate is a prior prediction based on prior knowledge and the other is based on the new data. The purpose is to combine these two estimates to get an even better estimate. The Kalman filter is very similar to a Bayesian approach in that a Bayesian approach uses prior and sampling information to form a posterior distribution. The following is the Kalman filter equation for a univariate time series:

$$F(t+1) = wX(t) + (1-w)F(t) \tag{47}$$

such that

$$w = \frac{\sigma_F^2}{\sigma_F^2 + \sigma_X^2} \tag{48}$$

so

$$F(t+1) = \frac{\sigma_F^2 X(t) + \sigma_X^2 F(t)}{\sigma_F^2 + \sigma_X^2} \tag{49}$$

The equation $F(t+1) = wX(t) + (1-w)F(t)$ is identical to simple exponential smoothing.

As the uncertainty of the future increase, so too will the value of the variance of $X$ $\sigma_X^2$ relative to the variance of $F$ $\sigma_F^2$. This forces the denominator of the equation for $w$ to increase and $w$ to decrease, putting more weight on $F(t)$ relative to $X(t)$. Now, $w$ is a variable that changes over time. The parameter $w(t)$ is similar to $\alpha(t)$ in adaptive response rate exponential smoothing but, $\alpha(t)$ is calculated using past data and $w(t)$ is calculated using variances.

### *5.8 Multiattribute Utility Theory (MAUT)*

Multiattribute utility theory (MAUT) can also be used to determine which external events are likely to trigger a change in the monthly number of changes in the subject of interest. MAUT attempts to explicitly and directly model a decision maker's behavior. In this case, the one person who decides whether to change the subject of interest is the decision maker. The analyst wants to model the decision maker's behavior via the following equation:

$$u(x_1, x_2, x_3, x_4, x_5) = w_1 u_1(x_1) + w_2 u_2(x_2) + w_3 u_3(x_3) + w_4 u_4(x_4) + w_5 u_5(x_5) \tag{50}$$

where $w_1 + w_2 + w_3 + w_4 + w_5 = 1$

Thus, an additive value function is assumed. It is also assumed that the attributes $x_1$,$x_2$,$x_3$,$x_4$, and $x_5$ are independent and that $u_i(x_i)$ is a one dimensional value function for all $i, i = 1, 2, \ldots, 5$.

There are five steps to measuring value functions:

1. Familiarize the decision maker (DM) to the concepts and techniques of value-function measurement.
2. Identify the appropriate value-decomposition form, $u(x)$.
3. Measure the component value function, $u_i(x_i)$.
4. Determine the $w_i's$.
5. Validate the consistency of $u(x)$ against the decision maker's observed rankings.

The fifth step is the most important and the most difficult step.

### 5.9 *Comparisons and Contrasts of Simple Exponential Smoothing, Adaptive Response Rate Exponential Smoothing, and the Kalman Filter*

There are arguments for and against using simple exponential smoothing and adaptive response rate exponential smoothing. When using simple exponential smoothing, the analyst is given the freedom to use any $\alpha$-value based upon the characteristics of the time series and the expectations of the intervention's effect. The $\alpha$-value for an adaptive response rate exponential smoothing system, however, is based purely upon the data. In other words, the analyst has no input into the value of the $\alpha$. Calculating the best $\alpha$-value for the simple exponential smoothing model is a lot of work. The adaptive response rate method deletes the dilemma of determining the optimal $\alpha$-value. Thus, when forecasts are being made for a large number of time series with many data points, such as the case with the $p_{ijk}$'s, it is suggested to use adaptive response rate exponential smoothing. When there is some insight into the effect of the intervention, the recommended approach is simple exponential smoothing using a compromise programming approach to determine the best $\alpha$-value.

The simple exponential smoothing approach does not account for changes in either the mean or the variance. The adaptive response rate exponential smoothing approach does not account for changes in the variance. The Kalman filter accounts for both changes in the mean and the variance simultaneously.

Classical statistical estimation, such as the simple exponential approach, attempts to minimize the mean squared error. Minimizing the mean squared error is an appropriate criterion for past observations but may not be for future predictions.

There are some major problems involving the Kalman filter that involve many technical questions not being answered satisfactorily. The approach is not well understood in the Operations Research career field because the approach has its beginnings in Engineering and is described in state-space notation. Initial estimates for the parameters and variances are difficult to calculate in the univariate case and are compounded by the need of initial estimates for covariances and the transition matrix in the multivariate case. Each update of the estimates using the Kalman filter requires the calculation of the variance of $X$, $\sigma_x^2$.

All three approaches update its estimates using new and old information. The weights combining new and old information for the exponential smoothing approaches are functions of past data. The weights for the Kalman filter are functions of variances. The estimates for all three approaches can be computed recursively.

There are definite advantages between simple exponential smoothing, adaptive response rate exponential smoothing, and Kalman filtering. Both of the exponential smoothing approaches are special cases of the Kalman filter. Thus, the best approach is a Kalman filter.

### *5.10 Comparisons and Contrasts of Intervention Analysis and the Kalman Filter*

Both intervention analysis and the Kalman filter are ways of accounting for an external event's effect on a time series. Intervention analysis has some advantages and some disadvantages over the Kalman filter. It becomes necessary to use a Kalman filter as opposed to intervention analysis when there are no experts in the field, the experts do not know the answers to the analyst's questions, the experts are not willing to tell the analyst the answers, or the analyst does not have the time or the resources. Experts in the field are the most important aspect of intervention analysis. If the analyst can not find any qualified experts, intervention analysis can not be conducted.

Intervention analysis tells you when the intervention occurred. The Kalman filter tells you approximately when an intervention occurs. The analyst would want to use Intervention Analysis if the time an intervention started is important. On the other hand, a Kalman filter can account for interventions without recognizing the particular incident.

Intervention analysis tells you what the intervention is, while Kalman filtering does not. Intervention analysis is more causal since you know what is causing the effect on the time series. With Kalman filtering, all you know is that there is an effect.

The Kalman filter is less labor intensive but is more difficult to understand.

A Kalman filter explains switches by use of the data in terms of the mean and the variance without having to recognize the time the incident occurs or the shape of the effect. Intervention analysis determines the switches exogenously from the data.

A Kalman filter continuously groups the data. For example, a Kalman filter may have truncated the data for the number of the subject of interest changes into five groups. The intervention analysis simply groups the data into two groups: one group is the data occurring before January 1991 and the other group is the data occurring after January 1991.

A Kalman filter uses past and present information contained in the data to calculate its estimates. Intervention analysis uses exogenous information along with past and present data observations to calculate its estimates. As a result, the Kalman filter has to dig out the information that it needs to calculate its estimates and quite often, this information is hard to dig out. Because the intervention analysis determines the effects of the interventions exogenously, there is no chance of modelling noise in the data as the intervention effect. There is a higher chance that the Kalman filter can pick up noise.

### *5.11 Comparisons and Contrasts Between Intervention Analysis, Social Judgement Theory, and Multiattribute Utility Theory*

The factor analysis that was conducted to determine which external events are potential interventions is also called social judgment theory (SJT). SJT is quite similar to Multi-Attribute Utility Theory (MAUT) except in the way that the input data are obtained (27:446). The objective of SJT is:

> to obtain an explicit, quantitative description of the decision maker's cognitive system (the policy), by which information is integrated in to an expression of preference. (27:446)

Social judgement theory uses linear regression to capture decision making.

The basic additive utility model from MAUT theory is:

$$u(x_1, x_2, x_3) = w_1 u_1(x_1) + w_2 u_2(x_2) + w_3 u_3(x_3) \tag{51}$$

Social judgement theory introduces the following transformation:

$$x_i = u_i(x_i) \tag{52}$$

Equation 51 can now be rewritten as:

$$u(x_1, x_2, x_3) = w_1x_1 + w_2x_2 + w_3x_3 \tag{53}$$

By introducing the transformation, it is no longer necessary to calculate the component utility functions but the weights still need to be calibrated.

In MAUT, the weights are determined by directly questioning experts. As a result, a set of independent and consistent equations are derived and the weights are computed (27:448). In SJT, the weights are calculated by analyzing past behavior.

The two techniques differ mainly in the approach for calculating the weights and the component utility functions. The standard lottery technique introduced by Keeney and Raiffa is used to calculate the weights and the utility functions when applying MAUT (27:455).

In 1976, Bertil Tell conducted a study where SJT and MAUT were compared. All MAUT type models produced higher correlation coefficients, higher parameter estimate variances, and more inconsistencies (27:456). The MAUT models also are harder to understand, harder to use, harder to explain, and more difficult for the experts to answer.

# *VI. Conclusions and Recommendations*

## *6.1 The Best and Most Parsimonious Univariate Model for Target Region 11*

The primary objective of this research was to develop a methodology to estimate predicted probabilities $\hat{p}_{ijk}$ of an event type $i$ occurring at geographical region $j$ at time block $k$. The problem was scaled down to only include event type 2 and time block 1. The four iterative stage of the Box-Jenkins forecasting approach was used to develop a univariate STARMA model for geographical region 11. The best and most parsimonious univariate STARMA model for target region is the SSTMA$(2_{1,1})_{12}$ model. The SSTMA$(2_{1,1})_{12}$ model used to fit geographical region 11 has a SSR of 2.37. After the negative probability predictions are transformed to zero probability predictions, the SSR drops to 2.34. If the relative analytical probabilities are used to fit geographical region 11, the SSR value is 3.06. Before performing transformations on the negative probability predictions, the SSTMA$(2_{1,1})_{12}$ model produced a SSR that is 22.55% better than that of the relative analytical probabilities. After performing the transformations on the negative proability predictions, the SSTMA$(2_{1,1})_{12}$ model produced a SSR that is 34.95% better than that of the relative analytical probabilities.

## *6.2 Aptness of the STARMA Model*

The univariate STARMA methodology is appropriate for predicting probabilities $\hat{p}_{ijk}$ in order to task for a world-wide sensor system. STARMA modelling is appropriate when temporal and spatial correlations are present in the data. A temporal correlation is present in the historical data and can be found in the three dimensional plots. There is a definite 12 month seasonality present in the historical data base. There is also a definite spatial relationship between the 22 regions in the historical data.

### *6.3 Causal and Correlative*

The model created is both a causal and a correlative model (8). The model is a correlative model in that both the temporal and the spatial correlations were accounted. The model is a causal model because it incorporates the analytical model probability estimates $\breve{p}_{2j1}$. The analytical model is a causal model that attempts to estimate the probabilities based on physical factors.

### *6.4 Analytical Model as a Simple Filter*

The analytical model was used, in essence, as a simple filter (8). The normalized analytical model estimates were subtracted from the historical probabilities in the same fashion that a filter is used. The normalized analytical model filtered the historical data by removing the portion of the historical relative frequencies described by the normalized analytical probability estimates.

### *6.5 Characteristics of the SSTMA$(2_{1,1,})_{12}$ Model*

The best and most parsimonious model selected is SSTMA$(2_{1,1})_{12}$. There are no AR terms in the best and most parsimonious model. This simply means that there is no trend or momentum in the data (8). The MA terms approximate the fidelity of the time series by continuous blocks of the data (8). The MA terms, in effect, take care of the deviations from the mean.

### *6.6 STARMA Software*

If more STARMA modelling is to be conducted on the historical data base and the analytical data base, it is imperative that some commercial software that performs either univariate STARMA modelling or full STARMA modelling be acquired.

### *6.7 Other Methods Besides STARMA*

### *6.8 Differencing to Remove Seasonality*

The SSTMA model was fit using the 12 period differenced data because the data after the 12 period difference appeared to be more "uniform" in the spatial dimension. A 12 period difference was applied to $NHA$, the difference between the historical data base and the normalized analytical data base, in an attempt to remove a 12 period seasonality. Unfortunately, the 12 period difference did not remove the 12 period seasonality as hoped. It was assumed that the seasonality present in the data may be non-stationary as a consequence of the expected 11 year cycle. This may have been a poor assumption because the 12 period difference did not remove the 12 period seasonality. It would be interesting to perform further analysis with the undifferenced data of the normalized historical data minus the analytical data$NHA$ and compare the results with those obtained in this research.

### *6.9 Robustness of the Univariate STARMA Model Selected*

The univariate STARMA model selected was developed using a combination ARMA modelling approach. The target Region selected was geographical region 11. The robustness of the univariate model developed was checked using target region 7. The target region 11 model was used to develop forecasts for target region 7. The probability predictions for target region 7 appear to be good fits, however the SSR value after performing the transformation on the negative probabilities was 4.47. This is a relatively high SSR value. The residual autocorrelations and partial autocorrelations appeared to be white noise.

The SSTMA$(2_{1,1})_{12}$ was estimated on the combined series of target region 11. The estimated model did not pass the diagnostic checking because the parameter coefficient $\theta_{2,1}$ was not significantly different from zero. An appropriate univariate STARMA model for target region 7 may be SSTMA$(2_{1,0})_{12}$. If this is the case, it would not be detrimental to model target region 7 as a

$SSTMA(2_{1,1})_{12}$. However, modelling target region 11 with a $SSTMA(2_{1,0})_{12}$ would be inappropriate.

The regions that were first order neighbors to either target region 7 or target region 11 appeared to share similar characteristics. Likewise, the regions that were second order neighbors to both target region 7 and target region 11 appear similar. But the first order neighbors and the second order neighbors did not appear to share many characteristics between them. As a result, it may not be appropriate to develop a full STARMA model on the data set. Perhaps, it would advantageous to develop two separate full STARMA models: one STARMA model on the first order neighbors and one STARMA model on the second order neighbors.

### *6.10 Intervention Analysis*

Intervention analysis may be an appropriate method for modelling the probabilities of an event type $i$ occurring at geographical region $j$ at time block $k$ is policy or doctrinal changes and other external events are triggering a gradual or abrupt change in the mean or tend of teh time series. The difficulty with intervention analysis is all factors must be determined exogenously from the data. Factor analysis, ARMA model building, and the fractile method are three techniques that can be used to determine what external events are interventions, when an intervention has occurred, and the function of the effect when an intervention occurs. The difficulty with the smoothing techniques is smoothing the noise while modeling the interventions. The difficulty with Kalman Filtering is understanding its principles. The difficulty with multiattribute theory is validating the consistency of the calibrated $u(x)$ function with the decision maker's actual behavior. The two best methods other than STARMA to solve the problem are intervention analysis and the Kalman filter. The degenerate form of the Kalman filter is regression. Thus, all regressions are a degenerate form of the Kalman filter.

# Appendix A. *Historical Database for Event Type 2 and Time Block 1*

This appendix contains monthly observations of the historical relative frequency $X_{ijk}$ for event type 2 and time block 1. The database covers all twenty-two geographical regions from January of 1985 through July of 1991. The database consists of 1,738 $X_{2,j,1}$'s. Each column represents a geographical region and each row represents a month.

Table 30. Historical Frequencies $X_{2,j,1}$

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| JAN 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.50 | 0.25 | 0.00 | 0.00 | 0.13 | 0.00 | 0.13 |
| FEB 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAR 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.17 | 0.33 | 0.33 | 0.00 | 0.00 | 0.00 |
| APR 1985 | 0.00 | 0.00 | 0.00 | 0.50 | 0.00 | 0.00 | 0.00 | 0.00 | 0.50 | 0.00 | 0.00 |
| MAY 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.67 | 0.00 |
| JUN 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.60 | 0.40 | 0.00 | 0.00 |
| JUL 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| AUG 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| SEP 1985 | 0.00 | 0.00 | 0.00 | 0.89 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| OCT 1985 | 0.00 | 0.00 | 0.21 | 0.00 | 0.00 | 0.21 | 0.18 | 0.11 | 0.14 | 0.04 | 0.00 |
| NOV 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.67 | 0.00 | 0.33 |
| DEC 1985 | 0.00 | 0.00 | 0.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| JAN 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.20 | 0.00 | 0.40 |
| FEB 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAR 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| APR 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAY 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.50 | 0.17 | 0.00 | 0.00 | 0.00 |
| JUN 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.11 | 0.00 |
| JUL 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| AUG 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| SEP 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| OCT 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| NOV 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| DEC 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| JAN 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| FEB 1987 | 0.00 | 0.00 | 0.00 | 0.28 | 0.17 | 0.39 | 0.06 | 0.06 | 0.00 | 0.00 | 0.00 |
| MAR 1987 | 0.00 | 0.00 | 0.00 | 0.10 | 0.26 | 0.14 | 0.30 | 0.16 | 0.00 | 0.03 | 0.01 |

Table 31. Historical Frequencies $X_{2,j,1}$ continued

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| APR 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.04 | 0.29 | 0.04 | 0.00 | 0.00 |
| MAY 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.67 | 0.22 | 0.00 |
| JUN 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.29 | 0.00 | 0.00 | 0.00 | 0.57 | 0.00 |
| JUL 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| AUG 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.06 | 0.05 | 0.38 | 0.05 | 0.03 |
| SEP 1987 | 0.00 | 0.00 | 0.00 | 0.03 | 0.00 | 0.05 | 0.43 | 0.11 | 0.08 | 0.08 | 0.11 |
| OCT 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.16 | 0.03 | 0.03 | 0.09 | 0.06 | 0.31 |
| NOV 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.05 | 0.05 | 0.30 | 0.16 | 0.08 | 0.05 | 0.05 |
| DEC 1987 | 0.00 | 0.03 | 0.00 | 0.00 | 0.00 | 0.03 | 0.17 | 0.10 | 0.24 | 0.00 | 0.17 |
| JAN 1988 | 0.00 | 0.02 | 0.00 | 0.02 | 0.14 | 0.12 | 0.24 | 0.12 | 0.14 | 0.00 | 0.07 |
| FEB 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.50 | 0.50 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAR 1988 | 0.00 | 0.00 | 0.08 | 0.00 | 0.00 | 0.08 | 0.67 | 0.00 | 0.18 | 0.00 | 0.00 |
| APR 1988 | 0.00 | 0.00 | 0.00 | 0.08 | 0.00 | 0.23 | 0.08 | 0.15 | 0.00 | 0.00 | 0.08 |
| MAY 1988 | 0.00 | 0.00 | 0.08 | 0.00 | 0.00 | 0.08 | 0.67 | 0.00 | 0.18 | 0.00 | 0.00 |
| JUN 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.50 |
| JUL 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.04 | 0.00 | 0.04 | 0.08 | 0.58 |
| AUG 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.06 | 0.02 | 0.09 | 0.00 | 0.11 |
| SEP 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.06 | 0.18 | 0.10 | 0.12 | 0.29 | 0.16 |
| OCT 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.04 | 0.01 | 0.44 | 0.37 | 0.04 |
| NOV 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.12 | 0.14 | 0.03 | 0.25 | 0.16 | 0.05 |
| DEC 1988 | 0.00 | 0.00 | 0.00 | 0.15 | 0.15 | 0.12 | 0.00 | 0.03 | 0.32 | 0.06 | 0.15 |
| JAN 1989 | 0.00 | 0.00 | 0.15 | 0.02 | 0.35 | 0.14 | 0.07 | 0.10 | 0.09 | 0.00 | 0.03 |
| FEB 1989 | 0.00 | 0.00 | 0.00 | 0.02 | 0.02 | 0.00 | 0.20 | 0.40 | 0.00 | 0.00 | 0.33 |
| MAR 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.15 | 0.03 | 0.38 | 0.00 | 0.06 | 0.00 | 0.00 |
| APR 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.10 | 0.01 | 0.15 | 0.01 | 0.02 |

Table 32. Historical Frequencies $X_{2,j,1}$ continued

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| MAY 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.01 | 0.00 | 0.01 |
| JUN 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.00 | 0.02 | 0.09 | 0.04 | 0.00 | 0.02 |
| JUL 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.02 | 0.02 | 0.03 | 0.01 | 0.01 |
| AUG 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.03 | 0.00 | 0.07 |
| SEP 1989 | 0.00 | 0.02 | 0.00 | 0.00 | 0.02 | 0.00 | 0.16 | 0.02 | 0.26 | 0.05 | 0.02 |
| OCT 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.20 | 0.00 | 0.00 | 0.28 | 0.04 |
| NOV 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.03 | 0.03 | 0.57 | 0.00 | 0.26 | 0.03 | 0.03 |
| DEC 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.20 | 0.00 | 0.00 | 0.80 | 0.00 |
| JAN 1990 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.05 | 0.65 | 0.22 | 0.00 | 0.00 | 0.00 |
| FEB 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.15 | 0.00 | 0.41 | 0.21 | 0.00 | 0.24 | 0.00 |
| MAR 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.09 | 0.14 | 0.21 | 0.29 | 0.02 |
| APR 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.29 | 0.41 | 0.00 |
| MAY 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.05 | 0.00 | 0.23 |
| JUN 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.01 | 0.01 | 0.24 |
| JUL 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.00 | 0.00 | 0.04 | 0.00 | 0.26 |
| AUG 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.08 | 0.05 | 0.00 | 0.13 | 0.00 | 0.53 |
| SEP 1990 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.74 |
| OCT 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.05 | 0.12 | 0.14 | 0.02 | 0.10 | 0.02 | 0.54 |
| NOV 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.87 | 0.09 | 0.00 | 0.00 | 0.00 |
| DEC 1990 | 0.00 | 0.10 | 0.10 | 0.00 | 0.00 | 0.00 | 0.70 | 0.00 | 0.00 | 0.10 | 0.00 |
| JAN 1991 | 0.00 | 0.00 | 0.00 | 0.00 | 0.08 | 0.00 | 0.88 | 0.00 | 0.02 | 0.00 | 0.02 |
| FEB 1991 | 0.00 | 0.00 | 0.00 | 0.00 | 0.04 | 0.00 | 0.46 | 0.02 | 0.00 | 0.00 | 0.38 |
| MAR 1991 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.27 | 0.02 | 0.25 | 0.13 | 0.08 |
| APR 1991 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.06 | 0.00 | 0.27 | 0.00 | 0.38 |
| MAY 1991 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.10 | 0.04 | 0.57 |
| JUN 1991 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.03 | 0.11 | 0.66 |
| JUL 1991 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.05 | 0.02 | 0.64 |

Table 33. Historical Frequencies $X_{2,j,1}$ continued

| | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| JAN 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| FEB 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAR 1985 | 0.00 | 0.00 | 0.00 | 0.17 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| APR 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAY 1985 | 0.00 | 0.33 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| JUN 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| JUL 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| AUG 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| SEP 1985 | 0.00 | 0.11 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| OCT 1985 | 0.00 | 0.07 | 0.00 | 0.00 | 0.00 | 0.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| NOV 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| DEC 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| JAN 1986 | 0.40 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| FEB 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAR 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| APR 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAY 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.33 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| JUN 1986 | 0.00 | 0.00 | 0.78 | 0.00 | 0.11 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| JUL 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| AUG 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| SEP 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| OCT 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| NOV 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| DEC 1986 | 0.25 | 0.50 | 0.00 | 0.00 | 0.25 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| JAN 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| FEB 1987 | 0.00 | 0.06 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAR 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

Table 34. Historical Frequencies $X_{2,j,1}$ continued

| | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| APR 1987 | 0.58 | 0.00 | 0.00 | 0.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAY 1987 | 0.00 | 0.06 | 0.06 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| JUN 1987 | 0.00 | 0.14 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| JUL 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| AUG 1987 | 0.08 | 0.13 | 0.13 | 0.02 | 0.00 | 0.03 | 0.05 | 0.00 | 0.00 | 0.00 | 0.00 |
| SEP 1987 | 0.05 | 0.03 | 0.00 | 0.00 | 0.00 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| OCT 1987 | 0.03 | 0.06 | 0.03 | 0.16 | 0.00 | 0.00 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 |
| NOV 1987 | 0.02 | 0.06 | 0.03 | 0.00 | 0.00 | 0.08 | 0.02 | 0.08 | 0.00 | 0.00 | 0.00 |
| DEC 1987 | 0.03 | 0.14 | 0.03 | 0.00 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| JAN 1988 | 0.05 | 0.00 | 0.05 | 0.00 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| FEB 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAR 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| APR 1988 | 0.08 | 0.31 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAY 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| JUN 1988 | 0.33 | 0.00 | 0.00 | 0.17 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| JUL 1988 | 0.06 | 0.14 | 0.06 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| AUG 1988 | 0.00 | 0.24 | 0.04 | 0.10 | 0.11 | 0.14 | 0.08 | 0.01 | 0.00 | 0.00 | 0.00 |
| SEP 1988 | 0.04 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| OCT 1988 | 0.07 | 0.00 | 0.01 | 0.00 | 0.01 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 |
| NOV 1988 | 0.14 | 0.01 | 0.00 | 0.10 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| DEC 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| JAN 1989 | 0.03 | 0.00 | 0.00 | 0.01 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| FEB 1989 | 0.00 | 0.00 | 0.00 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAR 1989 | 0.00 | 0.00 | 0.00 | 0.34 | 0.00 | 0.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| APR 1989 | 0.07 | 0.24 | 0.00 | 0.34 | 0.02 | 0.00 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 |

Table 35. Historical Frequencies $X_{2,j,1}$ continued

| | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| MAY 1989 | 0.06 | 0.01 | 0.00 | 0.47 | 0.14 | 0.18 | 0.06 | 0.04 | 0.00 | 0.00 | 0.00 |
| JUN 1989 | 0.01 | 0.37 | 0.02 | 0.18 | 0.13 | 0.01 | 0.00 | 0.07 | 0.00 | 0.00 | 0.00 |
| JUL 1989 | 0.01 | 0.16 | 0.00 | 0.52 | 0.01 | 0.00 | 0.13 | 0.08 | 0.00 | 0.00 | 0.00 |
| AUG 1989 | 0.17 | 0.21 | 0.00 | 0.28 | 0.17 | 0.03 | 0.00 | 0.00 | 0.00 | 0.03 | 0.00 |
| SEP 1989 | 0.14 | 0.17 | 0.00 | 0.05 | 0.03 | 0.00 | 0.00 | 0.05 | 0.02 | 0.00 | 0.00 |
| OCT 1989 | 0.24 | 0.08 | 0.00 | 0.00 | 0.04 | 0.12 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| NOV 1989 | 0.02 | 0.00 | 0.00 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| DEC 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| JAN 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.05 | 0.00 | 0.00 | 0.00 |
| FEB 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAR 1990 | 0.00 | 0.07 | 0.00 | 0.00 | 0.02 | 0.11 | 0.00 | 0.06 | 0.00 | 0.00 | 0.00 |
| APR 1990 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.12 | 0.06 | 0.10 | 0.00 | 0.00 | 0.00 |
| MAY 1990 | 0.04 | 0.09 | 0.02 | 0.09 | 0.00 | 0.03 | 0.34 | 0.11 | 0.00 | 0.00 | 0.00 |
| JUN 1990 | 0.00 | 0.02 | 0.00 | 0.12 | 0.00 | 0.04 | 0.47 | 0.07 | 0.00 | 0.00 | 0.00 |
| JUL 1990 | 0.06 | 0.06 | 0.04 | 0.06 | 0.02 | 0.13 | 0.09 | 0.21 | 0.00 | 0.00 | 0.00 |
| AUG 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.21 | 0.00 | 0.00 | 0.00 |
| SEP 1990 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.23 | 0.00 | 0.00 | 0.00 |
| OCT 1990 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.00 | 0.00 | 0.00 |
| NOV 1990 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.00 | 0.00 | 0.00 |
| DEC 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| JAN 1991 | 0.00 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| FEB 1991 | 0.00 | 0.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAR 1991 | 0.00 | 0.00 | 0.00 | 0.15 | 0.00 | 0.00 | 0.00 | 0.10 | 0.00 | 0.00 | 0.00 |
| APR 1991 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.29 | 0.00 | 0.00 | 0.00 |
| MAY 1991 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.27 | 0.00 | 0.00 | 0.00 |
| JUN 1991 | 0.00 | 0.00 | 0.01 | 0.05 | 0.00 | 0.00 | 0.00 | 0.14 | 0.00 | 0.00 | 0.00 |
| JUL 1991 | 0.00 | 0.06 | 0.00 | 0.13 | 0.01 | 0.00 | 0.00 | 0.10 | 0.00 | 0.00 | 0.00 |

## Appendix B. *Average Relative Frequencies Over Each Year for Event Type 2 and Time Block 1*

This appendix contains the average relative frequency $X_{ijk}$ observed over each year for event type 2 and time block 1 for all twenty-two geographical regions. Each column represents a year and each row represents a geographical region.

Table 36. Historical Frequencies $X_{2,j,1}$ Over Each Year

| | 1985 | 1986 | 1987 | 1988 | 1989 | 1990 | 1991 |
|---|---|---|---|---|---|---|---|
| 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 2 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 |
| 3 | 0.02 | 0.00 | 0.00 | 0.01 | 0.01 | 0.01 | 0.00 |
| 4 | 0.20 | 0.00 | 0.03 | 0.02 | 0.00 | 0.00 | 0.00 |
| 5 | 0.04 | 0.08 | 0.04 | 0.03 | 0.05 | 0.02 | 0.02 |
| 6 | 0.14 | 0.00 | 0.09 | 0.11 | 0.02 | 0.02 | 0.00 |
| 7 | 0.04 | 0.04 | 0.12 | 0.22 | 0.16 | 0.24 | 0.24 |
| 8 | 0.09 | 0.01 | 0.08 | 0.04 | 0.05 | 0.06 | 0.01 |
| 9 | 0.15 | 0.02 | 0.13 | 0.15 | 0.08 | 0.07 | 0.10 |
| 10 | 0.06 | 0.01 | 0.09 | 0.08 | 0.10 | 0.09 | 0.04 |
| 11 | 0.04 | 0.03 | 0.06 | 0.15 | 0.05 | 0.21 | 0.39 |
| 12 | 0.00 | 0.05 | 0.07 | 0.06 | 0.06 | 0.01 | 0.00 |
| 13 | 0.04 | 0.04 | 0.06 | 0.06 | 0.10 | 0.02 | 0.02 |
| 14 | 0.00 | 0.07 | 0.02 | 0.01 | 0.00 | 0.01 | 0.00 |
| 15 | 0.01 | 0.00 | 0.02 | 0.03 | 0.19 | 0.02 | 0.05 |
| 16 | 0.00 | 0.06 | 0.00 | 0.01 | 0.05 | 0.00 | 0.00 |
| 17 | 0.00 | 0.00 | 0.01 | 0.01 | 0.03 | 0.04 | 0.00 |
| 18 | 0.00 | 0.00 | 0.01 | 0.01 | 0.02 | 0.08 | 0.01 |
| 19 | 0.00 | 0.00 | 0.01 | 0.00 | 0.02 | 0.09 | 0.13 |
| 20 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 21 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 22 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

# Appendix C. *Average Relative Frequencies Over Each Season for Event Type 2 and Time Block 1*

This appendix contains the average relative frequency $X_{ijk}$ observed over each season for event type 2 and time block 1. The database covers all twenty-two geographical regions. Each column represents a season and each row represents a geographical region.

Table 37. Average Relative Frequencies $X_{2,j,1}$ Observed Over Each Season

| | WINTER | SPRING | SUMMER | FALL |
|---|---|---|---|---|
| REGION 1 | 0.00 | 0.00 | 0.00 | 0.00 |
| REGION 2 | 0.01 | 0.00 | 0.00 | 0.00 |
| REGION 3 | 0.01 | 0.01 | 0.00 | 0.01 |
| REGION 4 | 0.08 | 0.03 | 0.00 | 0.06 |
| REGION 5 | 0.09 | 0.07 | 0.00 | 0.01 |
| REGION 6 | 0.14 | 0.04 | 0.03 | 0.05 |
| REGION 7 | 0.25 | 0.18 | 0.02 | 0.22 |
| REGION 8 | 0.07 | 0.06 | 0.05 | 0.05 |
| REGION 9 | 0.06 | 0.15 | 0.08 | 0.17 |
| REGION 10 | 0.07 | 0.09 | 0.06 | 0.10 |
| REGION 11 | 0.09 | 0.07 | 0.24 | 0.16 |
| REGION 12 | 0.04 | 0.04 | 0.05 | 0.05 |
| REGION 13 | 0.04 | 0.06 | 0.10 | 0.04 |
| REGION 14 | 0.00 | 0.00 | 0.07 | 0.00 |
| REGION 15 | 0.00 | 0.08 | 0.11 | 0.02 |
| REGION 16 | 0.02 | 0.03 | 0.04 | 0.01 |
| REGION 17 | 0.00 | 0.02 | 0.03 | 0.02 |
| REGION 18 | 0.00 | 0.02 | 0.05 | 0.00 |
| REGION 19 | 0.00 | 0.05 | 0.06 | 0.03 |
| REGION 20 | 0.00 | 0.00 | 0.00 | 0.00 |
| REGION 21 | 0.00 | 0.00 | 0.00 | 0.00 |
| REGION 22 | 0.00 | 0.00 | 0.00 | 0.00 |

## Appendix D. *Analytical Database for Event Type 2 and Time Block 1*

This appendix contains monthly observations of the analytical predictions $\breve{p}_{ijk}$ for event type 2 and time block 1. The database covers all twenty-two geographical regions from January of 1985 through July of 1991. The database consists of 1,738 $\breve{p}_{2,j,1}$'s. Each column represents a geographical region and each row represents a month.

Table 38. Analytical Predictions $\breve{p}_{2,j,1}$

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| JAN 1985 | 0.01 | 0.00 | 0.00 | 0.01 | 0.02 | 0.03 | 0.09 | 0.08 | 0.04 | 0.01 | 0.00 |
| FEB 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.05 | 0.10 | 0.15 | 0.13 | 0.07 | 0.02 |
| MAR 1985 | 0.02 | 0.01 | 0.00 | 0.01 | 0.04 | 0.12 | 0.21 | 0.41 | 0.48 | 0.47 | 0.30 |
| APR 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.04 | 0.16 | 0.29 | 0.44 | 0.55 | 0.59 |
| MAY 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.09 | 0.18 | 0.30 | 0.35 | 0.42 |
| JUN 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.08 | 0.16 | 0.23 | 0.36 |
| JUL 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.06 | 0.13 | 0.20 | 0.30 | 0.50 |
| AUG 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.06 | 0.13 | 0.25 | 0.38 | 0.45 |
| SEP 1985 | 0.00 | 0.00 | 0.00 | 0.01 | 0.01 | 0.08 | 0.30 | 0.50 | 0.70 | 0.70 | 0.63 |
| OCT 1985 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.04 | 0.23 | 0.41 | 0.54 | 0.56 | 0.49 |
| NOV 1985 | 0.02 | 0.01 | 0.00 | 0.01 | 0.01 | 0.07 | 0.16 | 0.28 | 0.15 | 0.06 | 0.01 |
| DEC 1985 | 0.04 | 0.01 | 0.00 | 0.00 | 0.00 | 0.10 | 0.08 | 0.02 | 0.00 | 0.00 | 0.00 |
| JAN 1986 | 0.01 | 0.00 | 0.00 | 0.01 | 0.02 | 0.03 | 0.11 | 0.07 | 0.02 | 0.00 | 0.00 |
| FEB 1986 | 0.00 | 0.00 | 0.00 | 0.01 | 0.01 | 0.05 | 0.10 | 0.21 | 0.11 | 0.06 | 0.02 |
| MAR 1986 | 0.02 | 0.01 | 0.00 | 0.01 | 0.04 | 0.12 | 0.19 | 0.38 | 0.42 | 0.33 | 0.23 |
| APR 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.04 | 0.17 | 0.28 | 0.42 | 0.53 | 0.56 |
| MAY 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.09 | 0.18 | 0.29 | 0.33 | 0.40 |
| JUN 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.08 | 0.18 | 0.23 | 0.35 |
| JUL 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.06 | 0.12 | 0.19 | 0.30 | 0.47 |
| AUG 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.06 | 0.13 | 0.24 | 0.36 | 0.41 |
| SEP 1986 | 0.00 | 0.00 | 0.00 | 0.01 | 0.01 | 0.09 | 0.33 | 0.50 | 0.67 | 0.66 | 0.50 |
| OCT 1986 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.04 | 0.18 | 0.41 | 0.52 | 0.53 | 0.45 |
| NOV 1986 | 0.02 | 0.01 | 0.00 | 0.01 | 0.01 | 0.09 | 0.16 | 0.20 | 0.14 | 0.05 | 0.01 |
| DEC 1986 | 0.04 | 0.01 | 0.00 | 0.00 | 0.00 | 0.04 | 0.08 | 0.02 | 0.00 | 0.00 | 0.00 |
| JAN 1987 | 0.01 | 0.00 | 0.00 | 0.01 | 0.02 | 0.03 | 0.11 | 0.08 | 0.03 | 0.01 | 0.00 |
| FEB 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.05 | 0.10 | 0.15 | 0.13 | 0.07 | 0.02 |
| MAR 1987 | 0.02 | 0.01 | 0.00 | 0.01 | 0.03 | 0.12 | 0.23 | 0.42 | 0.51 | 0.46 | 0.34 |

Table 39. Analytical Predictions $\breve{p}_{2,j,1}$ continued

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| APR 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.04 | 0.15 | 0.31 | 0.48 | 0.57 | 0.62 |
| MAY 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.09 | 0.18 | 0.32 | 0.39 | 0.50 |
| JUN 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.07 | 0.19 | 0.27 | 0.41 |
| JUL 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.05 | 0.12 | 0.22 | 0.37 | 0.60 |
| AUG 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.05 | 0.14 | 0.28 | 0.49 | 0.60 |
| SEP 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.05 | 0.25 | 0.49 | 0.79 | 0.83 | 0.77 |
| OCT 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.04 | 0.21 | 0.42 | 0.59 | 0.65 | 0.64 |
| NOV 1987 | 0.01 | 0.00 | 0.00 | 0.00 | 0.01 | 0.07 | 0.24 | 0.33 | 0.40 | 0.30 | 0.18 |
| DEC 1987 | 0.01 | 0.00 | 0.00 | 0.00 | 0.01 | 0.03 | 0.13 | 0.18 | 0.11 | 0.04 | 0.01 |
| JAN 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.05 | 0.11 | 0.17 | 0.13 | 0.07 | 0.02 |
| FEB 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.05 | 0.11 | 0.21 | 0.26 | 0.23 | 0.15 |
| MAR 1988 | 0.01 | 0.00 | 0.00 | 0.01 | 0.03 | 0.16 | 0.41 | 0.57 | 0.66 | 0.68 | 0.69 |
| ARP 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.05 | 0.23 | 0.42 | 0.67 | 0.79 | 0.84 |
| MAY 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.06 | 0.18 | 0.38 | 0.68 | 0.93 |
| JUN 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.05 | 0.13 | 0.34 | 0.65 |
| JUL 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.06 | 0.21 | 0.54 | 0.86 |
| AUG 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.03 | 0.10 | 0.31 | 0.68 | 0.89 |
| SEP 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.14 | 0.33 | 0.74 | 0.97 | 1.00 |
| OCT 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.12 | 0.29 | 0.63 | 0.89 | 0.95 |
| NOV 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.06 | 0.26 | 0.49 | 0.67 | 0.75 | 0.75 |
| DEC 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.08 | 0.26 | 0.45 | 0.53 | 0.38 | 0.25 |
| JAN 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.04 | 0.13 | 0.27 | 0.40 | 0.27 | 0.18 |
| FEB 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.03 | 0.19 | 0.37 | 0.51 | 0.40 | 0.34 |
| MAR 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.07 | 0.26 | 0.52 | 0.77 | 0.91 | 0.95 |

Table 40. Analytical Predictions $\breve{p}_{2,j,1}$ continued

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| APR 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.10 | 0.24 | 0.53 | 0.85 | 0.97 |
| MAY 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.09 | 0.26 | 0.61 | 0.93 |
| JUN 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.10 | 0.27 | 0.59 |
| JUL 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.04 | 0.16 | 0.48 | 0.85 |
| AUG 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.06 | 0.26 | 0.60 | 0.90 |
| SEP 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.09 | 0.27 | 0.65 | 0.96 | 1.00 |
| OCT 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.13 | 0.29 | 0.64 | 0.89 | 0.95 |
| NOV 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.06 | 0.30 | 0.56 | 0.83 | 0.92 | 0.91 |
| DEC 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.08 | 0.25 | 0.45 | 0.55 | 0.41 | 0.30 |
| JAN 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.04 | 0.13 | 0.29 | 0.40 | 0.30 | 0.21 |
| FEB 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.03 | 0.19 | 0.39 | 0.53 | 0.42 | 0.36 |
| MAR 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.06 | 0.25 | 0.52 | 0.77 | 0.91 | 0.95 |
| APR 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.10 | 0.23 | 0.55 | 0.85 | 0.97 |
| MAY 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.10 | 0.27 | 0.63 | 0.93 |
| JUN 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.03 | 0.11 | 0.31 | 0.63 |
| JUL 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.05 | 0.18 | 0.52 | 0.86 |
| AUG 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.08 | 0.27 | 0.64 | 0.90 |
| SEP 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.11 | 0.31 | 0.68 | 0.97 | 1.00 |
| OCT 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.14 | 0.30 | 0.67 | 0.87 | 0.92 |
| NOV 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.05 | 0.25 | 0.47 | 0.69 | 0.78 | 0.78 |
| DEC 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.07 | 0.26 | 0.45 | 0.54 | 0.39 | 0.27 |
| JAN 1991 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.04 | 0.13 | 0.26 | 0.29 | 0.26 | 0.17 |
| FEB 1991 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.03 | 0.18 | 0.34 | 0.49 | 0.38 | 0.32 |
| MAR 1991 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.08 | 0.31 | 0.53 | 0.78 | 0.90 | 0.92 |
| APR 1991 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.12 | 0.28 | 0.57 | 0.86 | 0.96 |
| MAY 1991 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.05 | 0.12 | 0.30 | 0.64 | 0.94 |
| JUN 1991 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.03 | 0.12 | 0.31 | 0.64 |
| JUL 1991 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.05 | 0.19 | 0.52 | 0.86 |

Table 41. Analytical Predictions $\breve{p}_{2,j,1}$ continued

| | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| JAN 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| FEB 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAR 1985 | 0.18 | 0.08 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| APR 1985 | 0.58 | 0.49 | 0.39 | 0.26 | 0.13 | 0.04 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAY 1985 | 0.66 | 0.65 | 0.59 | 0.53 | 0.43 | 0.29 | 0.16 | 0.06 | 0.02 | 0.00 | 0.00 |
| JUN 1985 | 0.59 | 0.56 | 0.48 | 0.42 | 0.32 | 0.21 | 0.11 | 0.04 | 0.01 | 0.00 | 0.00 |
| JUL 1985 | 0.5 | 0.43 | 0.36 | 0.27 | 0.16 | 0.07 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 |
| AUG 1985 | 0.39 | 0.34 | 0.25 | 0.15 | 0.06 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| SEP 1985 | 0.42 | 0.25 | 0.08 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| OCT 1985 | 0.41 | 0.28 | 0.15 | 0.05 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| NOV 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| DEC 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| JAN 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| FEB 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAR 1986 | 0.12 | 0.04 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| APR 1986 | 0.52 | 0.43 | 0.32 | 0.19 | 0.07 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAY 1986 | 0.61 | 0.61 | 0.55 | 0.47 | 0.36 | 0.22 | 0.10 | 0.03 | 0.01 | 0.00 | 0.00 |
| JUN 1986 | 0.57 | 0.58 | 0.44 | 0.37 | 0.27 | 0.16 | 0.07 | 0.02 | 0.00 | 0.00 | 0.00 |
| JUL 1986 | 0.52 | 0.39 | 0.31 | 0.21 | 0.11 | 0.04 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 |
| AUG 1986 | 0.35 | 0.29 | 0.20 | 0.10 | 0.03 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| SEP 1986 | 0.36 | 0.18 | 0.05 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| OCT 1986 | 0.36 | 0.24 | 0.11 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| NOV 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| DEC 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| JAN 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| FEB 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAR 1987 | 0.23 | 0.11 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

Table 42. Analytical Predictions $\check{p}_{2,j,1}$ continued

| | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| APR 1987 | 0.64 | 0.65 | 0.48 | 0.37 | 0.23 | 0.10 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAY 1987 | 0.73 | 0.73 | 0.71 | 0.61 | 0.53 | 0.4 | 0.26 | 0.14 | 0.05 | 0.01 | 0.00 |
| JUN 1987 | 0.67 | 0.69 | 0.63 | 0.52 | 0.45 | 0.34 | 0.22 | 0.12 | 0.04 | 0.01 | 0.00 |
| JUL 1987 | 0.65 | 0.63 | 0.51 | 0.44 | 0.34 | 0.22 | 0.11 | 0.04 | 0.01 | 0.00 | 0.00 |
| AUG 1987 | 0.62 | 0.52 | 0.47 | 0.38 | 0.26 | 0.15 | 0.06 | 0.01 | 0.00 | 0.00 | 0.00 |
| SEP 1987 | 0.71 | 0.48 | 0.31 | 0.13 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| OCT 1987 | 0.56 | 0.48 | 0.35 | 0.2 | 0.07 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| NOV 1987 | 0.07 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| DEC 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| JAN 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| FEB 1988 | 0.07 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAR 1988 | 0.52 | 0.36 | 0.17 | 0.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| ARP 1988 | 0.85 | 0.83 | 0.84 | 0.77 | 0.61 | 0.44 | 0.23 | 0.07 | 0.01 | 0.00 | 0.00 |
| MAY 1988 | 0.98 | 0.98 | 0.97 | 0.92 | 0.88 | 0.84 | 0.7 | 0.55 | 0.35 | 0.17 | 0.05 |
| JUN 1988 | 0.9 | 0.94 | 0.93 | 0.9 | 0.84 | 0.82 | 0.73 | 0.65 | 0.54 | 0.41 | 0.27 |
| JUL 1988 | 0.96 | 0.97 | 0.95 | 0.86 | 0.84 | 0.8 | 0.65 | 0.41 | 0.15 | 0.02 | 0.00 |
| AUG 1988 | 0.94 | 0.94 | 0.88 | 0.84 | 0.79 | 0.6 | 0.34 | 0.09 | 0.01 | 0.00 | 0.00 |
| SEP 1988 | 1.00 | 0.99 | 0.94 | 0.86 | 0.64 | 0.43 | 0.19 | 0.03 | 0.00 | 0.00 | 0.00 |
| OCT 1988 | 0.95 | 0.91 | 0.86 | 0.73 | 0.66 | 0.57 | 0.45 | 0.3 | 0.16 | 0.06 | 0.02 |
| NOV 1988 | 0.74 | 0.58 | 0.48 | 0.34 | 0.19 | 0.07 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 |
| DEC 1988 | 0.10 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| JAN 1989 | 0.08 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| FEB 1989 | 0.24 | 0.13 | 0.04 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAR 1989 | 0.91 | 0.83 | 0.59 | 0.39 | 0.16 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| APR 1989 | 1.00 | 1.00 | 1.00 | 1.00 | 0.98 | 0.92 | 0.75 | 0.62 | 0.43 | 0.22 | 0.07 |

Table 43. Analytical Predictions $\breve{p}_{2,j,1}$ continued

| | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| MAY 1989 | 0.99 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 0.99 | 0.91 | 0.76 | 0.64 |
| JUN 1989 | 0.94 | 1.00 | 1.00 | 1.00 | 1.00 | 0.99 | 0.92 | 0.89 | 0.84 | 0.71 | 0.52 |
| JUL 1989 | 0.98 | 1.00 | 1.00 | 1.00 | 0.98 | 0.9 | 0.88 | 0.85 | 0.67 | 0.45 | 0.2 |
| AUG 1989 | 0.98 | 1.00 | 0.99 | 0.98 | 0.9 | 0.88 | 0.77 | 0.6 | 0.34 | 0.10 | 0.01 |
| SEP 1989 | 1.00 | 1.00 | 1.00 | 0.97 | 0.9 | 0.71 | 0.54 | 0.31 | 0.11 | 0.63 | 0.00 |
| OCT 1989 | 0.94 | 0.9 | 0.85 | 0.72 | 0.65 | 0.55 | 0.43 | 0.28 | 0.14 | 0.05 | 0.01 |
| NOV 1989 | 0.83 | 0.65 | 0.45 | 0.19 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| DEC 1989 | 0.15 | 0.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| JAN 1989 | 0.11 | 0.04 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| FEB 1990 | 0.26 | 0.15 | 0.06 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAR 1990 | 0.92 | 0.83 | 0.6 | 0.41 | 0.18 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| APR 1990 | 1.00 | 1.00 | 1.00 | 1.00 | 0.98 | 0.92 | 0.74 | 0.59 | 0.39 | 0.18 | 0.05 |
| MAY 1990 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 0.98 | 0.80 | 0.7 | 0.54 |
| JUN 1990 | 0.95 | 1.00 | 1.00 | 1.00 | 0.99 | 0.95 | 0.89 | 0.87 | 0.73 | 0.55 | 0.31 |
| JUL 1990 | 0.98 | 0.99 | 0.99 | 0.98 | 0.91 | 0.87 | 0.85 | 0.71 | 0.51 | 0.24 | 0.06 |
| AUG 1990 | 0.98 | 0.99 | 0.97 | 0.92 | 0.87 | 0.84 | 0.67 | 0.43 | 0.16 | 0.02 | 0.00 |
| SEP 1990 | 1.00 | 1.00 | 0.99 | 0.92 | 0.86 | 0.62 | 0.41 | 0.17 | 0.03 | 0.00 | 0.00 |
| OCT 1990 | 0.91 | 0.87 | 0.82 | 0.7 | 0.62 | 0.51 | 0.37 | 0.22 | 0.10 | 0.03 | 0.01 |
| NOV 1990 | 0.78 | 0.63 | 0.54 | 0.41 | 0.26 | 0.13 | 0.04 | 0.01 | 0.00 | 0.00 | 0.00 |
| DEC 1990 | 0.12 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| JAN 1991 | 0.08 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| FEB 1991 | 0.22 | 0.11 | 0.04 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAR 1991 | 0.87 | 0.79 | 0.51 | 0.29 | 0.08 | 0.64 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| APR 1991 | 0.99 | 1.00 | 1.00 | 0.98 | 0.94 | 0.88 | 0.67 | 0.48 | 0.26 | 0.08 | 0.01 |
| MAY 1991 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 0.99 | 0.96 | 0.91 | 0.74 | 0.6 | 0.41 |
| JUN 1991 | 0.96 | 1.00 | 1.00 | 1.00 | 0.98 | 0.88 | 0.87 | 0.79 | 0.65 | 0.42 | 0.17 |
| JUL 1991 | 0.97 | 0.99 | 0.98 | 0.94 | 0.86 | 0.85 | 0.74 | 0.56 | 0.29 | 0.07 | 0.01 |

## Appendix E. *Normalized Analytical Database for Event Type 2 and Time Block 1*

This appendix contains monthly observations of the normalized analytical predictions $\breve{p}_{ijk}$ for event type 2 and time block 1. The database covers all twenty-two geographical regions from January of 1985 through July of 1991. The database consists of 1,738 normalized $\breve{p}_{2,j,1}$'s. Each column represents a geographical region and each row represents a month.

Table 44. Normalized Analytical Predictions $\breve{p}_{2,j,1}$

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| JAN 1985 | 0.03 | 0.00 | 0.00 | 0.03 | 0.07 | 0.10 | 0.31 | 0.28 | 0.14 | 0.03 | 0.00 |
| FEB 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.09 | 0.19 | 0.28 | 0.25 | 0.13 | 0.04 |
| MAR 1985 | 0.01 | 0.00 | 0.00 | 0.00 | 0.02 | 0.05 | 0.09 | 0.17 | 0.20 | 0.20 | 0.13 |
| APR 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.04 | 0.07 | 0.11 | 0.14 | 0.15 |
| MAY 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.04 | 0.06 | 0.07 | 0.09 |
| JUN 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.02 | 0.04 | 0.06 | 0.10 |
| JUL 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.04 | 0.07 | 0.10 | 0.17 |
| AUG 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.05 | 0.10 | 0.15 | 0.18 |
| SEP 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.08 | 0.14 | 0.19 | 0.19 | 0.17 |
| OCT 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.07 | 0.13 | 0.17 | 0.18 | 0.15 |
| NOV 1985 | 0.03 | 0.01 | 0.00 | 0.01 | 0.01 | 0.09 | 0.21 | 0.36 | 0.19 | 0.08 | 0.01 |
| DEC 1985 | 0.16 | 0.04 | 0.00 | 0.00 | 0.00 | 0.40 | 0.32 | 0.08 | 0.00 | 0.00 | 0.00 |
| JAN 1986 | 0.04 | 0.00 | 0.00 | 0.04 | 0.07 | 0.11 | 0.41 | 0.26 | 0.07 | 0.00 | 0.00 |
| FEB 1986 | 0.00 | 0.00 | 0.00 | 0.02 | 0.02 | 0.09 | 0.18 | 0.37 | 0.19 | 0.11 | 0.04 |
| MAR 1986 | 0.01 | 0.01 | 0.00 | 0.01 | 0.02 | 0.06 | 0.10 | 0.20 | 0.22 | 0.17 | 0.12 |
| APR 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.05 | 0.08 | 0.12 | 0.15 | 0.16 |
| MAY 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.04 | 0.07 | 0.08 | 0.09 |
| JUN 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.02 | 0.05 | 0.07 | 0.10 |
| JUL 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.04 | 0.07 | 0.11 | 0.17 |
| AUG 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.03 | 0.06 | 0.11 | 0.16 | 0.19 |
| SEP 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.03 | 0.10 | 0.15 | 0.20 | 0.20 | 0.15 |
| OCT 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.06 | 0.14 | 0.18 | 0.18 | 0.16 |
| NOV 1986 | 0.03 | 0.01 | 0.00 | 0.01 | 0.01 | 0.13 | 0.23 | 0.29 | 0.20 | 0.07 | 0.01 |
| DEC 1986 | 0.21 | 0.05 | 0.00 | 0.00 | 0.00 | 0.21 | 0.42 | 0.11 | 0.00 | 0.00 | 0.00 |
| JAN 1987 | 0.03 | 0.00 | 0.00 | 0.03 | 0.07 | 0.10 | 0.37 | 0.27 | 0.10 | 0.03 | 0.00 |
| FEB 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.09 | 0.19 | 0.28 | 0.25 | 0.13 | 0.04 |
| MAR 1987 | 0.01 | 0.00 | 0.00 | 0.00 | 0.01 | 0.05 | 0.09 | 0.17 | 0.20 | 0.18 | 0.13 |

Table 45. Normalized Analytical Predictions $\check{p}_{2,j,1}$ continued

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| APR 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.03 | 0.07 | 0.10 | 0.12 | 0.13 |
| MAY 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.03 | 0.06 | 0.07 | 0.09 |
| JUN 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.04 | 0.06 | 0.09 |
| JUL 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.03 | 0.05 | 0.09 | 0.14 |
| AUG 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.03 | 0.07 | 0.12 | 0.15 |
| SEP 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.05 | 0.10 | 0.16 | 0.17 | 0.16 |
| OCT 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.05 | 0.10 | 0.14 | 0.15 | 0.15 |
| NOV 1987 | 0.01 | 0.00 | 0.00 | 0.00 | 0.01 | 0.04 | 0.15 | 0.20 | 0.25 | 0.19 | 0.11 |
| DEC 1987 | 0.02 | 0.00 | 0.00 | 0.00 | 0.02 | 0.06 | 0.25 | 0.35 | 0.21 | 0.08 | 0.02 |
| JAN 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.09 | 0.20 | 0.30 | 0.23 | 0.13 | 0.04 |
| FEB 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.05 | 0.10 | 0.19 | 0.23 | 0.21 | 0.14 |
| MAR 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.04 | 0.10 | 0.13 | 0.15 | 0.16 | 0.16 |
| ARP 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.03 | 0.05 | 0.09 | 0.10 | 0.11 |
| MAY 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.02 | 0.04 | 0.07 | 0.10 |
| JUN 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.01 | 0.04 | 0.07 |
| JUL 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.03 | 0.07 | 0.10 |
| AUG 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.04 | 0.09 | 0.12 |
| SEP 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.04 | 0.09 | 0.12 | 0.12 |
| OCT 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.03 | 0.07 | 0.10 | 0.11 |
| NOV 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.05 | 0.09 | 0.12 | 0.14 | 0.14 |
| DEC 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.04 | 0.13 | 0.22 | 0.25 | 0.18 | 0.12 |
| JAN 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.03 | 0.09 | 0.19 | 0.29 | 0.19 | 0.13 |
| FEB 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.08 | 0.16 | 0.23 | 0.18 | 0.15 |
| MAR 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.04 | 0.08 | 0.12 | 0.14 | 0.15 |
| APR 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.02 | 0.05 | 0.08 | 0.09 |

Table 46. Normalized Analytical Predictions $\check{p}_{2,j,1}$ continued

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| MAY 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.02 | 0.05 | 0.08 |
| JUN 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.03 | 0.05 |
| JUL 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.05 | 0.08 |
| AUG 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.03 | 0.06 | 0.10 |
| SEP 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.03 | 0.06 | 0.09 | 0.10 |
| OCT 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.03 | 0.08 | 0.11 | 0.11 |
| NOV 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.05 | 0.10 | 0.14 | 0.16 | 0.16 |
| DEC 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.04 | 0.11 | 0.20 | 0.25 | 0.18 | 0.13 |
| JAN 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.03 | 0.08 | 0.19 | 0.26 | 0.19 | 0.14 |
| FEB 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.08 | 0.16 | 0.22 | 0.18 | 0.15 |
| MAR 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.04 | 0.08 | 0.12 | 0.14 | 0.15 |
| APR 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.02 | 0.05 | 0.08 | 0.09 |
| MAY 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.02 | 0.05 | 0.08 |
| JUN 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.03 | 0.06 |
| JUL 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.02 | 0.05 | 0.09 |
| AUG 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.03 | 0.07 | 0.10 |
| SEP 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.03 | 0.07 | 0.11 | 0.11 |
| OCT 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.04 | 0.08 | 0.11 | 0.11 |
| NOV 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.04 | 0.08 | 0.12 | 0.13 | 0.13 |
| DEC 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.03 | 0.12 | 0.21 | 0.25 | 0.18 | 0.13 |
| JAN 1991 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.03 | 0.10 | 0.21 | 0.23 | 0.21 | 0.13 |
| FEB 1991 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.08 | 0.16 | 0.23 | 0.18 | 0.15 |
| MAR 1991 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.05 | 0.08 | 0.12 | 0.13 | 0.14 |
| APR 1991 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.03 | 0.06 | 0.09 | 0.10 |
| MAY 1991 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.03 | 0.05 | 0.08 |
| JUN 1991 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.03 | 0.07 |
| JUL 1991 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.02 | 0.06 | 0.10 |

Table 47. Normalized Analytical Predictions $\breve{p}_{2,j,1}$ continued

| | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| JAN 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| FEB 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAR 1985 | 0.08 | 0.03 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| APR 1985 | 0.15 | 0.12 | 0.10 | 0.07 | 0.03 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAY 1985 | 0.14 | 0.14 | 0.12 | 0.11 | 0.09 | 0.06 | 0.03 | 0.01 | 0.00 | 0.00 | 0.00 |
| JUN 1985 | 0.16 | 0.16 | 0.13 | 0.12 | 0.09 | 0.06 | 0.03 | 0.01 | 0.00 | 0.00 | 0.00 |
| JUL 1985 | 0.17 | 0.14 | 0.12 | 0.09 | 0.05 | 0.02 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 |
| AUG 1985 | 0.16 | 0.14 | 0.10 | 0.06 | 0.02 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| SEP 1985 | 0.11 | 0.07 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| OCT 1985 | 0.13 | 0.09 | 0.05 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| NOV 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| DEC 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| JAN 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| FEB 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAR 1986 | 0.06 | 0.02 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| APR 1986 | 0.15 | 0.12 | 0.09 | 0.05 | 0.02 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAY 1986 | 0.14 | 0.14 | 0.13 | 0.11 | 0.08 | 0.05 | 0.02 | 0.01 | 0.00 | 0.00 | 0.00 |
| JUN 1986 | 0.17 | 0.17 | 0.13 | 0.11 | 0.08 | 0.05 | 0.02 | 0.01 | 0.00 | 0.00 | 0.00 |
| JUL 1986 | 0.19 | 0.14 | 0.11 | 0.08 | 0.04 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| AUG 1986 | 0.16 | 0.13 | 0.09 | 0.05 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| SEP 1986 | 0.11 | 0.05 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| OCT 1986 | 0.13 | 0.08 | 0.04 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| NOV 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| DEC 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| JAN 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| FEB 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAR 1987 | 0.09 | 0.04 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

Table 48. Normalized Analytical Predictions $\breve{p}_{2,j,1}$ continued

| | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| APR 1987 | 0.14 | 0.14 | 0.10 | 0.08 | 0.05 | 0.02 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAY 1987 | 0.13 | 0.13 | 0.13 | 0.11 | 0.09 | 0.07 | 0.05 | 0.02 | 0.01 | 0.00 | 0.00 |
| JUN 1987 | 0.14 | 0.15 | 0.14 | 0.11 | 0.10 | 0.07 | 0.05 | 0.03 | 0.01 | 0.00 | 0.00 |
| JUL 1987 | 0.15 | 0.15 | 0.12 | 0.10 | 0.08 | 0.05 | 0.03 | 0.01 | 0.00 | 0.00 | 0.00 |
| AUG 1987 | 0.15 | 0.13 | 0.12 | 0.09 | 0.06 | 0.04 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 |
| SEP 1987 | 0.15 | 0.10 | 0.06 | 0.03 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| OCT 1987 | 0.13 | 0.11 | 0.08 | 0.05 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| NOV 1987 | 0.04 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| DEC 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| JAN 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| FEB 1988 | 0.06 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAR 1988 | 0.12 | 0.08 | 0.04 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| ARP 1988 | 0.11 | 0.11 | 0.11 | 0.10 | 0.08 | 0.06 | 0.03 | 0.01 | 0.00 | 0.00 | 0.00 |
| MAY 1988 | 0.10 | 0.10 | 0.10 | 0.10 | 0.09 | 0.09 | 0.07 | 0.06 | 0.04 | 0.02 | 0.01 |
| JUN 1988 | 0.10 | 0.10 | 0.10 | 0.10 | 0.09 | 0.09 | 0.08 | 0.07 | 0.06 | 0.05 | 0.03 |
| JUL 1988 | 0.12 | 0.12 | 0.11 | 0.10 | 0.10 | 0.10 | 0.08 | 0.05 | 0.02 | 0.00 | 0.00 |
| AUG 1988 | 0.13 | 0.13 | 0.12 | 0.11 | 0.11 | 0.08 | 0.05 | 0.01 | 0.00 | 0.00 | 0.00 |
| SEP 1988 | 0.12 | 0.12 | 0.11 | 0.10 | 0.08 | 0.05 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 |
| OCT 1988 | 0.11 | 0.11 | 0.10 | 0.09 | 0.08 | 0.07 | 0.05 | 0.04 | 0.02 | 0.01 | 0.00 |
| NOV 1988 | 0.14 | 0.11 | 0.09 | 0.06 | 0.04 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| DEC 1988 | 0.05 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| JAN 1989 | 0.06 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| FEB 1989 | 0.11 | 0.06 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAR 1989 | 0.14 | 0.13 | 0.09 | 0.06 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| APR 1989 | 0.09 | 0.09 | 0.09 | 0.09 | 0.09 | 0.09 | 0.07 | 0.06 | 0.04 | 0.02 | 0.01 |

Table 49. Normalized Analytical Predictions $\breve{p}_{2,j,1}$ continued

| | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| MAY 1989 | 0.08 | 0.08 | 0.08 | 0.08 | 0.08 | 0.08 | 0.08 | 0.08 | 0.07 | 0.06 | 0.05 |
| JUN 1989 | 0.09 | 0.09 | 0.09 | 0.09 | 0.09 | 0.09 | 0.09 | 0.08 | 0.08 | 0.07 | 0.05 |
| JUL 1989 | 0.09 | 0.10 | 0.10 | 0.10 | 0.09 | 0.09 | 0.08 | 0.08 | 0.06 | 0.04 | 0.02 |
| AUG 1989 | 0.10 | 0.11 | 0.11 | 0.10 | 0.10 | 0.09 | 0.08 | 0.06 | 0.04 | 0.01 | 0.00 |
| SEP 1989 | 0.10 | 0.10 | 0.10 | 0.10 | 0.09 | 0.07 | 0.05 | 0.03 | 0.01 | 0.06 | 0.00 |
| OCT 1989 | 0.11 | 0.11 | 0.10 | 0.09 | 0.08 | 0.07 | 0.05 | 0.03 | 0.02 | 0.01 | 0.00 |
| NOV 1989 | 0.14 | 0.11 | 0.08 | 0.03 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| DEC 1989 | 0.07 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| JAN 1990 | 0.07 | 0.03 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| FEB 1990 | 0.11 | 0.06 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAR 1990 | 0.14 | 0.13 | 0.09 | 0.06 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| APR 1990 | 0.09 | 0.09 | 0.09 | 0.09 | 0.09 | 0.09 | 0.07 | 0.06 | 0.04 | 0.02 | 0.00 |
| MAY 1990 | 0.08 | 0.08 | 0.08 | 0.08 | 0.08 | 0.08 | 0.08 | 0.08 | 0.07 | 0.06 | 0.05 |
| JUN 1990 | 0.09 | 0.10 | 0.10 | 0.10 | 0.10 | 0.09 | 0.09 | 0.08 | 0.07 | 0.05 | 0.03 |
| JUL 1990 | 0.10 | 0.10 | 0.10 | 0.10 | 0.09 | 0.09 | 0.09 | 0.07 | 0.05 | 0.02 | 0.01 |
| AUG 1990 | 0.11 | 0.11 | 0.11 | 0.11 | 0.10 | 0.10 | 0.08 | 0.05 | 0.02 | 0.00 | 0.00 |
| SEP 1990 | 0.11 | 0.11 | 0.11 | 0.10 | 0.09 | 0.07 | 0.05 | 0.02 | 0.00 | 0.00 | 0.00 |
| OCT 1990 | 0.11 | 0.11 | 0.10 | 0.09 | 0.08 | 0.06 | 0.05 | 0.03 | 0.01 | 0.00 | 0.00 |
| NOV 1990 | 0.13 | 0.11 | 0.09 | 0.07 | 0.04 | 0.02 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 |
| DEC 1990 | 0.06 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| JAN 1991 | 0.06 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| FEB 1991 | 0.10 | 0.05 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAR 1991 | 0.13 | 0.12 | 0.08 | 0.04 | 0.01 | 0.10 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| APR 1991 | 0.10 | 0.10 | 0.10 | 0.10 | 0.09 | 0.09 | 0.07 | 0.05 | 0.03 | 0.01 | 0.00 |
| MAY 1991 | 0.09 | 0.09 | 0.09 | 0.09 | 0.09 | 0.08 | 0.08 | 0.08 | 0.06 | 0.05 | 0.04 |
| JUN 1991 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.09 | 0.09 | 0.08 | 0.07 | 0.04 | 0.02 |
| JUL 1991 | 0.11 | 0.11 | 0.11 | 0.11 | 0.10 | 0.10 | 0.08 | 0.06 | 0.03 | 0.01 | 0.00 |

## Appendix F. *Database of NHA for Event Type 2 and Time Block 1*

This appendix contains monthly observations of the normalized analytical predictions $\check{p}_{ijk}$ subtracted from the historical observed frequencies $X_{ijk}$ for event type 2 and time block 1. The database covers all twenty-two geographical regions from January of 1985 through July of 1991. The database consists of 1,738 $NHA_{2,j,1}$'s. Each column represents a geographical region and each row represents a month.

Table 50. $NHA_{2,j,1}$

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| JAN 1985 | -0.03 | 0.00 | 0.00 | -0.03 | 0.43 | 0.15 | -0.31 | -0.28 | -0.01 | -0.03 | 0.13 |
| FEB 1985 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | 0. 1991 | -0.19 | -0.28 | -0.25 | -0.13 | -0.04 |
| MAR 1985 | -0.01 | 0.00 | 0.00 | 0.00 | -0.02 | 0.12 | 0.24 | 0.16 | -0.20 | -0.20 | -0.13 |
| APR 1985 | 0.00 | 0.00 | 0.00 | 0.50 | 0.00 | -0.01 | -0.04 | -0.07 | 0.39 | -0.14 | -0.15 |
| MAY 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | -0.04 | -0.06 | 0.60 | -0.09 |
| JUN 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.58 | 0.36 | -0.06 | -0.10 |
| JUL 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | -0.04 | -0.07 | -0.10 | -0.17 |
| AUG 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | -0.05 | -0.10 | -0.15 | -0.18 |
| SEP 1985 | 0.00 | 0.00 | 0.00 | 0.89 | 0.00 | -0.02 | -0.08 | -0.14 | -0.19 | -0.19 | -0.17 |
| OCT 1985 | 0.00 | 0.00 | 0.21 | 0.00 | 0.00 | 0.20 | 0.11 | -0.02 | -0.03 | -0.14 | -0.15 |
| NOV 1985 | -0.03 | -0.01 | 0.00 | -0.01 | -0.01 | -0.09 | -0.21 | -0.36 | 0.48 | -0.08 | 0.32 |
| DEC 1985 | -0.16 | -0.04 | 0.00 | 1.00 | 0.00 | -0.40 | -0.32 | -0.08 | 0.00 | 0.00 | 0.00 |
| JAN 1986 | -0.04 | 0.00 | 0.00 | -0.04 | -0.07 | -0.11 | -0.41 | -0.26 | 0.13 | 0.00 | 0.40 |
| FEB 1986 | 0.00 | 0.00 | 0.00 | -0.02 | -0.02 | -0.09 | -0.18 | -0.37 | -0.19 | -0.11 | -0.04 |
| MAR 1986 | -0.01 | -0.01 | 0.00 | -0.01 | 0.98 | -0.06 | -0.10 | -0.20 | -0.22 | -0.17 | -0.12 |
| APR 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | -0.05 | -0.08 | -0.12 | -0.15 | -0.16 |
| MAY 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.48 | 0.13 | -0.07 | -0.08 | -0.09 |
| JUN 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | -0.02 | -0.05 | 0.04 | -0.10 |
| JUL 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | -0.04 | -0.07 | -0.11 | -0.17 |
| AUG 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 | -0.06 | -0.11 | -0.16 | -0.19 |
| SEP 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 | -0.10 | -0.15 | -0.20 | -0.20 | -0.15 |
| OCT 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | -0.06 | -0.14 | -0.18 | -0.18 | -0.16 |
| NOV 1986 | -0.03 | -0.01 | 0.00 | -0.01 | -0.01 | -0.13 | -0.23 | -0.29 | -0.20 | -0.07 | -0.01 |
| DEC 1986 | -0.21 | -0.05 | 0.00 | 0.00 | 0.00 | -0.21 | -0.42 | -0.11 | 0.00 | 0.00 | 0.00 |
| JAN 1987 | -0.03 | 0.00 | 0.00 | -0.03 | -0.07 | -0.10 | -0.37 | -0.27 | -0.10 | -0.03 | 0.00 |
| FEB 1987 | 0.00 | 0.00 | 0.00 | 0.28 | 0.15 | 0.30 | -0.13 | -0.22 | -0.25 | -0.13 | -0.04 |
| MAR 1987 | -0.01 | 0.00 | 0.00 | 0.10 | 0.25 | 0.09 | 0.21 | -0.01 | -0.20 | -0.15 | -0.12 |

Table 51. $NHA_{2,j,1}$ continued

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| APR 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.01 | 0.22 | -0.06 | -0.12 | -0.13 |
| MAY 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | -0.03 | 0.61 | 0.15 | -0.09 |
| JUN 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.29 | 0.00 | -0.02 | -0.04 | 0.51 | -0.09 |
| JUL 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | -0.03 | -0.05 | -0.09 | -0.14 |
| AUG 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.05 | 0.02 | 0.31 | -0.07 | -0.12 |
| SEP 1987 | 0.00 | 0.00 | 0.00 | 0.03 | 0.00 | 0.04 | 0.38 | 0.01 | -0.08 | -0.09 | -0.05 |
| OCT 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.15 | -0.02 | -0.07 | -0.05 | -0.09 | 0.16 |
| NOV 1987 | -0.01 | 0.00 | 0.00 | 0.00 | 0.04 | 0.01 | 0.15 | -0.04 | -0.17 | -0.14 | -0.06 |
| DEC 1987 | -0.02 | 0.03 | 0.00 | 0.00 | -0.02 | -0.03 | -0.08 | -0.25 | 0.03 | -0.08 | 0.15 |
| JAN 1988 | 0.00 | 0.02 | 0.00 | 0.02 | 0.12 | 0.03 | 0.04 | -0.18 | -0.09 | -0.13 | 0.03 |
| FEB 1988 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.45 | 0.40 | -0.19 | -0.23 | -0.21 | -0.14 |
| MAR 1988 | 0.00 | 0.00 | 0.08 | 0.00 | -0.01 | 0.04 | 0.57 | -0.13 | 0.03 | -0.16 | -0.16 |
| ARP 1988 | 0.00 | 0.00 | 0.00 | 0.08 | 0.00 | 0.22 | 0.05 | 0.10 | -0.09 | -0.10 | -0.03 |
| MAY 1988 | 0.00 | 0.00 | 0.08 | 0.00 | 0.00 | 0.08 | 0.66 | -0.02 | 0.14 | -0.07 | -0.10 |
| JUN 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | -0.01 | -0.04 | 0.43 |
| JUL 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.04 | -0.01 | 0.01 | 0.01 | 0.48 |
| AUG 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.06 | 0.01 | 0.05 | -0.09 | -0.01 |
| SEP 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.06 | 0.16 | 0.06 | 0.03 | 0.17 | 0.04 |
| OCT 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.03 | -0.02 | 0.37 | 0.27 | -0.07 |
| NOV 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.11 | 0.09 | -0.06 | 0.13 | 0.02 | -0.09 |
| DEC 1988 | 0.00 | 0.00 | 0.00 | 0.15 | 0.15 | 0.08 | -0.13 | -0.19 | 0.07 | -0.12 | 0.03 |
| JAN 1989 | 0.00 | 0.00 | 0.15 | 0.02 | 0.34 | 0.11 | -0.02 | -0.09 | -0.20 | -0.19 | -0.10 |
| FEB 1989 | 0.00 | 0.00 | 0.00 | 0.02 | 0.02 | -0.01 | 0.12 | 0.24 | -0.23 | -0.18 | 0.18 |
| MAR 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.15 | 0.02 | 0.34 | -0.08 | -0.06 | -0.14 | -0.15 |
| APR 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.09 | -0.01 | 0.10 | -0.07 | -0.07 |

Table 52. $NHA_{2,j,1}$ continued

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| MAY 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | -0.01 | -0.01 | -0.05 | -0.07 |
| JUN 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.00 | 0.02 | 0.09 | 0.03 | -0.03 | -0.03 |
| JUL 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.02 | 0.02 | 0.01 | -0.04 | -0.07 |
| AUG 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 | -0.06 | -0.03 |
| SEP 1989 | 0.00 | 0.02 | 0.00 | 0.00 | 0.02 | 0.00 | 0.15 | -0.01 | 0.20 | -0.04 | -0.08 |
| OCT 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.18 | -0.03 | -0.08 | 0.17 | -0.07 |
| NOV 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.03 | 0.02 | 0.52 | -0.10 | 0.12 | -0.13 | -0.13 |
| DEC 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 | 0.09 | -0.20 | -0.25 | 0.62 | -0.13 |
| JAN 1990 | 0.03 | 0.00 | 0.00 | 0.00 | -0.01 | 0.02 | 0.57 | 0.03 | -0.26 | -0.19 | -0.14 |
| FEB 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.15 | -0.01 | 0.33 | 0.05 | -0.22 | 0.07 | -0.15 |
| MAR 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.05 | 0.06 | 0.09 | 0.15 | -0.13 |
| APR 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | 0.24 | 0.33 | -0.09 |
| MAY 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.03 | -0.05 | 0.15 |
| JUN 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | -0.02 | 0.18 |
| JUL 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.00 | -0.01 | 0.02 | -0.05 | 0.17 |
| AUG 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.08 | 0.05 | -0.01 | 0.10 | -0.07 | 0.43 |
| SEP 1990 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | -0.03 | -0.06 | -0.11 | 0.63 |
| OCT 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.05 | 0.12 | 0.12 | -0.02 | 0.02 | -0.09 | 0.43 |
| NOV 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.83 | 0.01 | -0.12 | -0.13 | -0.13 |
| DEC 1990 | 0.00 | 0.10 | 0.10 | 0.00 | 0.00 | -0.03 | 0.58 | -0.21 | -0.25 | -0.08 | -0.13 |
| JAN 1991 | 0.00 | 0.00 | 0.00 | 0.00 | 0.07 | -0.03 | 0.78 | -0.21 | -0.21 | -0.21 | -0.11 |
| FEB 1991 | 0.00 | 0.00 | 0.00 | 0.00 | 0.04 | -0.01 | 0.38 | -0.14 | -0.23 | -0.18 | 0.23 |
| MAR 1991 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.22 | -0.06 | 0.13 | 0.00 | -0.06 |
| APR 1991 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.05 | -0.03 | 0.21 | -0.09 | 0.28 |
| MAY 1991 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | -0.01 | 0.07 | -0.01 | 0.49 |
| JUN 1991 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.08 | 0.59 |
| JUL 1991 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.03 | -0.04 | 0.54 |

Table 53. $NHA_{2,j,1}$ continued

| | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| JAN 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| FEB 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAR 1985 | -0.08 | -0.03 | -0.01 | 0.17 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| APR 1985 | -0.15 | -0.12 | -0.10 | -0.07 | -0.03 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAY 1985 | -0.14 | 0.19 | -0.12 | -0.11 | -0.09 | -0.06 | -0.03 | -0.01 | 0.00 | 0.00 | 0.00 |
| JUN 1985 | -0.16 | -0.16 | -0.13 | -0.12 | -0.09 | -0.06 | -0.03 | -0.01 | 0.00 | 0.00 | 0.00 |
| JUL 1985 | -0.17 | -0.14 | -0.12 | -0.09 | -0.05 | -0.02 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 |
| AUG 1985 | -0.16 | -0.14 | -0.10 | -0.06 | -0.02 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| SEP 1985 | -0.11 | 0.04 | -0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| OCT 1985 | -0.13 | -0.02 | -0.05 | -0.02 | 0.00 | 0.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| NOV 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| DEC 1985 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| JAN 1986 | 0.40 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| FEB 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAR 1986 | -0.06 | -0.02 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| APR 1986 | -0.15 | -0.12 | -0.09 | -0.05 | -0.02 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAY 1986 | -0.14 | -0.14 | -0.13 | -0.11 | 0.25 | -0.05 | -0.02 | -0.01 | 0.00 | 0.00 | 0.00 |
| JUN 1986 | -0.17 | -0.17 | 0.65 | -0.11 | 0.03 | -0.05 | -0.02 | -0.01 | 0.00 | 0.00 | 0.00 |
| JUL 1986 | -0.19 | -0.14 | -0.11 | -0.08 | -0.04 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| AUG 1986 | -0.16 | -0.13 | -0.09 | -0.05 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| SEP 1986 | -0.11 | -0.05 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| OCT 1986 | -0.13 | -0.08 | -0.04 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| NOV 1986 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| DEC 1986 | 0.25 | 0.50 | 0.00 | 0.00 | 0.25 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| JAN 1987 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| FEB 1987 | 0.00 | 0.06 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAR 1987 | -0.09 | -0.04 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

Table 54. $NHA_{2,j,1}$ continued

| | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| APR 1987 | 0.44 | -0.14 | -0.10 | -0.04 | -0.05 | -0.02 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAY 1987 | -0.13 | -0.07 | -0.07 | -0.11 | -0.09 | -0.07 | -0.05 | -0.02 | -0.01 | 0.00 | 0.00 |
| JUN 1987 | -0.14 | -0.01 | -0.14 | -0.11 | -0.10 | -0.07 | -0.05 | -0.03 | -0.01 | 0.00 | 0.00 |
| JUL 1987 | -0.15 | -0.15 | -0.12 | -0.10 | -0.08 | -0.05 | -0.03 | -0.01 | 0.00 | 0.00 | 0.00 |
| AUG 1987 | -0.07 | 0.00 | 0.01 | -0.07 | -0.06 | -0.01 | 0.04 | 0.00 | 0.00 | 0.00 | 0.00 |
| SEP 1987 | -0.10 | -0.07 | -0.06 | -0.03 | -0.01 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| OCT 1987 | -0.10 | -0.05 | -0.05 | 0.11 | -0.02 | 0.00 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 |
| NOV 1987 | -0.02 | 0.05 | 0.03 | 0.00 | 0.00 | 0.08 | 0.02 | 0.08 | 0.00 | 0.00 | 0.00 |
| DEC 1987 | 0.03 | 0.14 | 0.03 | 0.00 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| JAN 1988 | 0.05 | 0.00 | 0.05 | 0.00 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| FEB 1988 | -0.06 | -0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAR 1988 | -0.12 | -0.08 | -0.04 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| ARP 1988 | -0.03 | 0.20 | -0.11 | -0.10 | -0.08 | -0.06 | -0.03 | -0.01 | 0.00 | 0.00 | 0.00 |
| MAY 1988 | -0.10 | -0.10 | -0.10 | -0.10 | -0.09 | -0.09 | -0.07 | -0.06 | -0.04 | -0.02 | -0.01 |
| JUN 1988 | 0.23 | -0.10 | -0.10 | 0.07 | -0.09 | -0.09 | -0.08 | -0.07 | -0.06 | -0.05 | -0.03 |
| JUL 1988 | -0.06 | 0.02 | -0.05 | -0.10 | -0.10 | -0.10 | -0.08 | -0.05 | -0.02 | 0.00 | 0.00 |
| AUG 1988 | -0.13 | 0.11 | -0.08 | -0.01 | 0.00 | 0.06 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 |
| SEP 1988 | -0.08 | -0.10 | -0.11 | -0.10 | -0.08 | -0.05 | -0.02 | 0.00 | 0.00 | 0.00 | 0.00 |
| OCT 1988 | -0.04 | -0.11 | -0.09 | -0.09 | -0.07 | -0.07 | -0.04 | -0.04 | -0.02 | -0.01 | 0.00 |
| NOV 1988 | 0.00 | -0.10 | -0.09 | 0.04 | -0.04 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| DEC 1988 | -0.05 | -0.01 | 0.00 | 0.00 | 0.00 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| JAN 1989 | -0.03 | -0.01 | 0.00 | 0.01 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| FEB 1989 | -0.11 | -0.06 | -0.02 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAR 1989 | -0.14 | -0.13 | -0.09 | 0.28 | -0.03 | 0.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| APR 1989 | -0.02 | 0.15 | -0.09 | 0.25 | -0.07 | -0.09 | -0.04 | -0.06 | -0.04 | -0.02 | -0.01 |

Table 55. $NHA_{2,j,1}$ continued

| | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| MAY 1989 | -0.02 | -0.07 | -0.08 | 0.39 | 0.06 | 0.10 | -0.02 | -0.04 | -0.07 | -0.06 | -0.05 |
| JUN 1989 | -0.08 | 0.28 | -0.07 | 0.09 | 0.04 | -0.08 | -0.09 | -0.01 | -0.08 | -0.07 | -0.05 |
| JUL 1989 | -0.08 | 0.06 | -0.10 | 0.42 | -0.08 | -0.09 | 0.05 | 0.00 | -0.06 | -0.04 | -0.02 |
| AUG 1989 | 0.07 | 0.10 | -0.11 | 0.18 | 0.07 | -0.06 | -0.08 | -0.06 | -0.04 | 0.02 | 0.00 |
| SEP 1989 | 0.04 | 0.07 | -0.10 | -0.05 | -0.06 | -0.07 | -0.05 | 0.02 | 0.01 | -0.06 | 0.00 |
| OCT 1989 | 0.13 | -0.03 | -0.10 | -0.09 | -0.04 | 0.05 | -0.05 | -0.03 | -0.02 | -0.01 | 0.00 |
| NOV 1989 | -0.12 | -0.11 | -0.08 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| DEC 1989 | -0.07 | -0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| JAN 1990 | -0.07 | -0.03 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.05 | 0.00 | 0.00 | 0.00 |
| FEB 1990 | -0.11 | -0.06 | -0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAR 1990 | -0.14 | -0.06 | -0.09 | -0.06 | -0.01 | 0.11 | 0.00 | 0.06 | 0.00 | 0.00 | 0.00 |
| APR 1990 | -0.08 | -0.09 | -0.09 | -0.09 | -0.09 | 0.03 | -0.01 | 0.04 | -0.04 | -0.02 | 0.00 |
| MAY 1990 | -0.04 | 0.01 | -0.06 | 0.01 | -0.08 | -0.05 | 0.26 | 0.03 | -0.07 | -0.06 | -0.05 |
| JUN 1990 | -0.09 | -0.08 | -0.10 | 0.02 | -0.10 | -0.05 | 0.38 | -0.01 | -0.07 | -0.05 | -0.03 |
| JUL 1990 | -0.04 | -0.04 | -0.06 | -0.04 | -0.07 | 0.04 | 0.00 | 0.14 | -0.05 | -0.02 | -0.01 |
| AUG 1990 | -0.11 | -0.11 | -0.11 | -0.11 | -0.10 | -0.10 | -0.08 | 0.16 | -0.02 | 0.00 | 0.00 |
| SEP 1990 | -0.10 | -0.11 | -0.11 | -0.10 | -0.09 | -0.07 | -0.05 | 0.21 | 0.00 | 0.00 | 0.00 |
| OCT 1990 | -0.11 | -0.10 | -0.10 | -0.09 | -0.08 | -0.06 | -0.05 | -0.01 | -0.01 | 0.00 | 0.00 |
| NOV 1990 | -0.11 | -0.11 | -0.09 | -0.07 | -0.04 | -0.02 | -0.01 | 0.02 | 0.00 | 0.00 | 0.00 |
| DEC 1990 | -0.06 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| JAN 1991 | -0.06 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| FEB 1991 | -0.10 | -0.01 | -0.02 | 0.00 | 0.00 | 0.00 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 |
| MAR 1991 | -0.13 | -0.12 | -0.08 | 0.11 | -0.01 | -0.10 | 0.00 | 0.10 | 0.00 | 0.00 | 0.00 |
| APR 1991 | -0.10 | -0.10 | -0.10 | -0.10 | -0.09 | -0.09 | -0.07 | 0.24 | -0.03 | -0.01 | 0.00 |
| MAY 1991 | -0.09 | -0.09 | -0.09 | -0.09 | -0.09 | -0.08 | -0.08 | 0.19 | -0.06 | -0.05 | -0.04 |
| JUN 1991 | -0.10 | -0.10 | -0.09 | -0.05 | -0.10 | -0.09 | -0.09 | 0.06 | -0.07 | -0.04 | -0.02 |
| JUL 1991 | -0.11 | -0.05 | -0.11 | 0.02 | -0.09 | -0.10 | -0.08 | 0.04 | -0.03 | -0.01 | 0.00 |

# Appendix G. *Autocorrelations of* $NHA_{2,j,1}$

This appendix contains the autocorrelations of $NHA_{2,j,1}$ for all twenty-two geographical regions. The autocorrelations were calculated for the first twenty lags. Each column represents a geographical region and each row represents a lag. Any autocorrelation that is greater than 0.33 or less than -0.33 is significantly different from zero.

Table 56. Autocorrelations of $NHA_{2,j,1}$

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| LAG 1 | 0.28 | 0.09 | -0.06 | -0.07 | -0.02 | 0.24 | 0.49 | 0.25 | 0.15 | 0.13 | 0.56 |
| LAG 2 | -0.01 | 0.00 | 0.02 | -0.07 | -0.09 | 0.01 | 0.37 | 0.09 | 0.02 | 0.07 | 0.32 |
| LAG 3 | 0.00 | 0.02 | -0.06 | 0.36 | -0.04 | 0.02 | 0.16 | 0.06 | -0.18 | -0.09 | 0.06 |
| LAG 4 | -0.04 | 0.00 | -0.06 | -0.07 | -0.07 | 0.10 | 0.03 | -0.11 | -0.20 | 0.03 | -0.04 |
| LAG 5 | -0.05 | 0.00 | -0.06 | 0.16 | -0.06 | -0.03 | -0.02 | -0.27 | 0.06 | -0.04 | -0.05 |
| LAG 6 | -0.05 | 0.00 | -0.06 | -0.05 | -0.06 | -0.03 | 0.04 | -0.18 | -0.12 | 0.07 | -0.07 |
| LAG 7 | -0.05 | 0.00 | -0.06 | -0.05 | -0.06 | 0.03 | -0.04 | -0.11 | 0.14 | -0.05 | 0.02 |
| LAG 8 | -0.03 | 0.01 | 0.09 | 0.19 | -0.05 | 0.21 | 0.01 | -0.22 | -0.18 | -0.14 | 0.16 |
| LAG 9 | 0.00 | 0.03 | -0.06 | -0.04 | -0.07 | -0.03 | 0.03 | -0.01 | -0.16 | -0.23 | 0.21 |
| LAG 10 | -0.01 | 0.00 | 0.11 | -0.04 | -0.10 | -0.20 | 0.24 | 0.12 | -0.06 | -0.18 | 0.20 |
| LAG 11 | 0.23 | 0.03 | -0.04 | -0.06 | 0.10 | -0.02 | 0.26 | 0.24 | 0.11 | -0.07 | 0.14 |
| LAG 12 | 0.53 | 0.04 | -0.04 | -0.02 | 0.10 | 0.04 | 0.35 | 0.32 | 0.21 | 0.08 | 0.11 |
| LAG 13 | 0.12 | -0.05 | -0.04 | -0.04 | -0.07 | -0.08 | 0.15 | 0.20 | 0.07 | 0.04 | 0.05 |
| LAG 14 | -0.01 | -0.01 | -0.04 | 0.12 | 0.27 | -0.15 | 0.11 | 0.09 | -0.11 | -0.02 | -0.04 |
| LAG 15 | -0.01 | 0.12 | -0.04 | 0.03 | -0.05 | -0.11 | -0.08 | -0.04 | -0.20 | 0.04 | -0.13 |
| LAG 16 | -0.03 | 0.00 | -0.04 | -0.04 | -0.05 | -0.05 | -0.04 | -0.07 | -0.04 | 0.04 | -0.11 |
| LAG 17 | -0.04 | 0.00 | -0.04 | 0.10 | -0.05 | -0.06 | -0.05 | -0.18 | 0.00 | 0.16 | -0.13 |
| LAG 18 | -0.04 | 0.00 | -0.04 | 0.02 | -0.04 | -0.11 | 0.04 | -0.12 | 0.16 | 0.15 | -0.10 |
| LAG 19 | -0.04 | 0.00 | -0.04 | -0.03 | -0.04 | -0.04 | -0.01 | -0.24 | -0.03 | 0.01 | -0.08 |
| LAG 20 | -0.04 | 0.02 | -0.04 | -0.03 | -0.02 | -0.04 | 0.11 | -0.18 | -0.12 | -0.08 | -0.06 |

Table 57. Autocorrelations of $NHA_{2,j,1}$ continued

| | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| LAG 1 | 0.20 | 0.12 | 0.02 | 0.55 | 0.34 | 0.25 | 0.45 | 0.61 | 0.74 | 0.48 | 0.52 |
| LAG 2 | 0.10 | 0.15 | -0.01 | 0.43 | 0.20 | 0.15 | -0.02 | 0.34 | 0.35 | 0.15 | 0.07 |
| LAG 3 | -0.11 | -0.01 | 0.08 | 0.23 | 0.15 | 0.02 | -0.12 | 0.10 | 0.05 | 0.04 | -0.08 |
| LAG 4 | 0.05 | -0.01 | -0.02 | 0.18 | -0.04 | -0.09 | -0.11 | 0.15 | -0.09 | 0.03 | -0.11 |
| LAG 5 | -0.17 | -0.17 | 0.01 | 0.02 | -0.10 | -0.08 | -0.08 | 0.05 | -0.17 | 0.07 | -0.11 |
| LAG 6 | -0.29 | -0.14 | 0.02 | -0.05 | -0.04 | -0.24 | -0.01 | 0.16 | -0.15 | -0.13 | -0.11 |
| LAG 7 | -0.22 | -0.27 | 0.01 | -0.11 | 0.14 | -0.19 | -0.03 | 0.27 | -0.16 | -0.06 | -0.11 |
| LAG 8 | -0.17 | -0.01 | -0.01 | -0.07 | -0.06 | -0.24 | 0.02 | 0.41 | -0.11 | 0.02 | -0.11 |
| LAG 9 | -0.08 | -0.04 | 0.06 | -0.06 | 0.09 | 0.14 | -0.03 | 0.28 | 0.03 | -0.02 | -0.11 |
| LAG 10 | 0.05 | 0.04 | -0.05 | -0.07 | 0.06 | 0.09 | -0.08 | 0.21 | 0.30 | 0.09 | 0.05 |
| LAG 11 | 0.22 | 0.10 | 0.06 | -0.03 | 0.02 | 0.13 | -0.06 | 0.05 | 0.58 | 0.44 | 0.46 |
| LAG 12 | 0.16 | 0.18 | -0.01 | -0.08 | 0.06 | 0.09 | -0.20 | 0.12 | 0.73 | 0.59 | 0.71 |
| LAG 13 | 0.10 | 0.03 | -0.03 | -0.01 | 0.02 | 0.14 | -0.17 | 0.07 | 0.55 | 0.36 | 0.32 |
| LAG 14 | 0.22 | 0.08 | 0.08 | -0.06 | 0.03 | 0.25 | -0.06 | 0.09 | 0.25 | -0.01 | -0.01 |
| LAG 15 | 0.23 | -0.09 | 0.02 | -0.17 | 0.01 | -0.08 | 0.04 | 0.01 | -0.03 | 0.00 | -0.08 |
| LAG 16 | -0.02 | 0.05 | 0.00 | -0.16 | -0.08 | -0.04 | 0.04 | -0.01 | -0.12 | -0.06 | -0.09 |
| LAG 17 | -0.13 | -0.11 | 0.07 | -0.09 | -0.12 | -0.26 | 0.00 | -0.07 | -0.15 | -0.13 | -0.09 |
| LAG 18 | -0.09 | -0.10 | -0.01 | -0.05 | -0.14 | -0.14 | -0.02 | -0.04 | -0.15 | -0.12 | -0.09 |
| LAG 19 | -0.08 | 0.11 | 0.01 | 0.02 | -0.12 | -0.11 | -0.02 | -0.05 | -0.14 | -0.07 | -0.09 |
| LAG 20 | -0.16 | 0.03 | -0.02 | -0.03 | -0.01 | -0.13 | -0.07 | -0.07 | -0.12 | 0.01 | -0.10 |

## Appendix H. *NHAS Database for Event Type 2 and Time Block 1*

This appendix contains monthly observations of $NHAS_{2,j,1}$ for all twenty-two geographical regions. Each column represents a geographical region and each row represents a period.

Table 58. $NHAS_{2,j,1}$

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| PERIOD 1 | 0.00 | 0.00 | 0.00 | 0.00 | -0.51 | -0.26 | -0.10 | 0.02 | 0.13 | 0.03 | 0.27 |
| PERIOD 2 | 0.00 | 0.00 | 0.00 | -0.02 | 0.00 | -0.99 | 0.01 | -0.09 | 0.05 | 0.03 | 0.00 |
| PERIOD 3 | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 | -0.18 | -0.34 | -0.35 | -0.01 | 0.03 | 0.01 |
| PERIOD 4 | 0.00 | 0.00 | 0.00 | -0.50 | 0.00 | 0.00 | -0.01 | -0.01 | -0.51 | -0.01 | -0.01 |
| PERIOD 5 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.50 | 0.17 | 0.00 | -0.67 | -0.01 |
| PERIOD 6 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.60 | -0.41 | 0.11 | 0.00 |
| PERIOD 7 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | -0.01 |
| PERIOD 8 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | -0.01 | -0.01 | -0.01 |
| PERIOD 9 | 0.00 | 0.00 | 0.00 | -0.89 | 0.00 | -0.01 | -0.02 | -0.01 | -0.01 | -0.01 | 0.02 |
| PERIOD 10 | 0.00 | 0.00 | -0.21 | 0.00 | 0.00 | -0.21 | -0.17 | -0.12 | -0.15 | -0.05 | 0.00 |
| PERIOD 11 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 | -0.02 | 0.07 | -0.68 | 0.01 | -0.33 |
| PERIOD 12 | -0.05 | -0.01 | 0.00 | -1.00 | 0.00 | 0.19 | -0.10 | -0.03 | 0.00 | 0.00 | 0.00 |
| PERIOD 13 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.01 | 0.04 | -0.01 | -0.23 | -0.03 | -0.40 |
| PERIOD 14 | 0.00 | 0.00 | 0.00 | 0.30 | 0.17 | 0.38 | 0.05 | 0.15 | -0.05 | -0.03 | 0.00 |
| PERIOD 15 | 0.00 | 0.00 | 0.00 | 0.10 | -0.73 | 0.15 | 0.31 | 0.19 | 0.02 | 0.02 | -0.01 |
| PERIOD 16 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.06 | 0.30 | 0.06 | 0.03 | 0.02 |
| PERIOD 17 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.49 | -0.16 | 0.68 | 0.23 | 0.01 |
| PERIOD 18 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.29 | 0.00 | 0.01 | 0.01 | 0.47 | 0.02 |
| PERIOD 19 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.02 | 0.02 | 0.02 | 0.03 |
| PERIOD 20 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.08 | 0.07 | 0.42 | 0.09 | 0.07 |
| PERIOD 21 | 0.00 | 0.00 | 0.00 | 0.03 | 0.00 | 0.07 | 0.48 | 0.16 | 0.12 | 0.11 | 0.10 |
| PERIOD 22 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.16 | 0.04 | 0.07 | 0.13 | 0.09 | 0.31 |
| PERIOD 23 | 0.02 | 0.01 | 0.00 | 0.01 | 0.06 | 0.14 | 0.38 | 0.24 | 0.03 | -0.06 | -0.05 |
| PERIOD 24 | 0.19 | 0.08 | 0.00 | 0.00 | -0.02 | 0.18 | 0.34 | -0.14 | 0.03 | -0.08 | 0.15 |
| PERIOD 25 | 0.03 | 0.02 | 0.00 | 0.05 | 0.19 | 0.13 | 0.41 | 0.08 | 0.01 | -0.09 | 0.03 |
| PERIOD 26 | 0.00 | 0.00 | 0.00 | -0.28 | -0.16 | 0.16 | 0.53 | 0.03 | 0.01 | -0.08 | -0.10 |
| PERIOD 27 | 0.01 | 0.00 | 0.08 | -0.10 | -0.26 | -0.05 | 0.37 | -0.13 | 0.23 | -0.01 | -0.04 |
| PERIOD 28 | 0.00 | 0.00 | 0.00 | 0.08 | 0.00 | 0.23 | 0.04 | -0.13 | -0.03 | 0.02 | 0.10 |
| PERIOD 29 | 0.00 | 0.00 | 0.08 | 0.00 | 0.00 | 0.08 | 0.68 | 0.01 | -0.47 | -0.22 | -0.01 |
| PERIOD 30 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.29 | 0.00 | 0.01 | 0.03 | -0.55 | 0.52 |
| PERIOD 31 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.05 | 0.02 | 0.07 | 0.10 | 0.62 |
| PERIOD 32 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | 0.01 | -0.01 | -0.26 | -0.02 | 0.11 |
| PERIOD 33 | 0.00 | 0.00 | 0.00 | -0.03 | 0.02 | 0.02 | -0.22 | 0.05 | 0.11 | 0.26 | 0.09 |

Table 59. $NHAS_{2,j,1}$ continued

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| PERIOD 34 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.15 | 0.05 | 0.05 | 0.42 | 0.36 | -0.23 |
| PERIOD 35 | 0.01 | 0.00 | 0.00 | 0.00 | -0.05 | 0.10 | -0.06 | -0.02 | 0.29 | 0.16 | -0.03 |
| PERIOD 36 | 0.02 | -0.03 | 0.00 | 0.15 | 0.16 | 0.11 | -0.05 | 0.06 | 0.04 | -0.05 | -0.12 |
| PERIOD 37 | 0.00 | -0.02 | 0.15 | 0.00 | 0.22 | 0.08 | -0.07 | 0.09 | -0.10 | -0.07 | -0.13 |
| PERIOD 38 | 0.00 | 0.00 | 0.00 | 0.02 | 0.03 | -0.47 | -0.28 | 0.43 | 0.01 | 0.03 | 0.31 |
| PERIOD 39 | 0.00 | 0.00 | -0.08 | 0.00 | 0.16 | -0.02 | -0.24 | 0.05 | -0.09 | 0.02 | 0.01 |
| PERIOD 40 | 0.00 | 0.00 | 0.00 | -0.08 | 0.00 | -0.23 | 0.04 | -0.11 | 0.19 | 0.03 | -0.04 |
| PERIOD 41 | 0.00 | 0.00 | -0.08 | 0.00 | 0.00 | -0.07 | -0.67 | 0.01 | -0.15 | 0.02 | 0.03 |
| PERIOD 42 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.00 | 0.02 | 0.10 | 0.05 | 0.01 | -0.46 |
| PERIOD 43 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | -0.02 | 0.02 | 0.00 | -0.05 | -0.55 |
| PERIOD 44 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.06 | -0.01 | -0.05 | 0.03 | -0.02 |
| PERIOD 45 | 0.00 | 0.02 | 0.00 | 0.00 | 0.00 | -0.06 | -0.01 | -0.07 | 0.17 | -0.22 | -0.12 |
| PERIOD 46 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.16 | -0.01 | -0.44 | -0.09 | 0.00 |
| PERIOD 47 | 0.00 | 0.00 | 0.00 | 0.00 | 0.03 | -0.09 | 0.43 | -0.04 | -0.01 | -0.15 | -0.04 |
| PERIOD 48 | 0.00 | 0.00 | 0.00 | -0.15 | -0.15 | -0.12 | 0.21 | -0.01 | -0.31 | 0.74 | -0.16 |
| PERIOD 49 | 0.03 | 0.00 | -0.15 | -0.02 | -0.35 | -0.09 | 0.59 | 0.12 | -0.06 | 0.00 | -0.04 |
| PERIOD 50 | 0.00 | 0.00 | 0.00 | -0.02 | 0.13 | 0.00 | 0.21 | -0.19 | 0.00 | 0.24 | -0.33 |
| PERIOD 51 | 0.00 | 0.00 | 0.00 | 0.00 | -0.15 | -0.03 | -0.29 | 0.14 | 0.15 | 0.29 | 0.02 |
| PERIOD 52 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.09 | -0.01 | 0.14 | 0.40 | -0.02 |
| PERIOD 53 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.04 | 0.00 | 0.22 |
| PERIOD 54 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | 0.01 | -0.02 | -0.09 | -0.03 | 0.01 | 0.21 |
| PERIOD 55 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.02 | -0.02 | -0.02 | 0.01 | -0.02 | 0.24 |
| PERIOD 56 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.08 | 0.05 | 0.00 | 0.10 | -0.01 | 0.45 |
| PERIOD 57 | 0.02 | -0.02 | 0.00 | 0.00 | -0.02 | 0.00 | -0.16 | -0.03 | -0.26 | -0.06 | 0.71 |
| PERIOD 58 | 0.00 | 0.00 | 0.00 | 0.00 | 0.05 | 0.12 | -0.06 | 0.02 | 0.09 | -0.26 | 0.50 |
| PERIOD 59 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 | -0.03 | 0.31 | 0.11 | -0.23 | 0.00 | -0.01 |
| PERIOD 60 | 0.00 | 0.10 | 0.10 | 0.00 | 0.00 | 0.00 | 0.49 | -0.01 | -0.01 | -0.70 | 0.01 |
| PERIOD 61 | -0.03 | 0.00 | 0.00 | 0.00 | 0.08 | -0.06 | 0.21 | -0.24 | 0.05 | -0.01 | 0.02 |
| PERIOD 62 | 0.00 | 0.00 | 0.00 | 0.00 | -0.11 | 0.00 | 0.04 | -0.19 | -0.01 | -0.24 | 0.38 |
| PERIOD 63 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.17 | -0.12 | 0.04 | -0.15 | 0.07 |
| PERIOD 64 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.05 | -0.01 | -0.02 | -0.41 | 0.38 |
| PERIOD 65 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.05 | 0.04 | 0.34 |
| PERIOD 66 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.02 | 0.10 | 0.42 |
| PERIOD 67 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | 0.00 | 0.00 | 0.01 | 0.02 | 0.37 |

Table 60. $NHAS_{2,j,1}$ continued

| | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| PERIOD 1 | 0.40 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| PERIOD 2 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| PERIOD 3 | 0.01 | 0.01 | 0.00 | -0.17 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| PERIOD 4 | 0.00 | 0.00 | 0.01 | 0.01 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| PERIOD 5 | 0.00 | -0.34 | 0.00 | 0.00 | 0.34 | 0.01 | 0.01 | 0.01 | 0.00 | 0.00 | 0.00 |
| PERIOD 6 | -0.01 | -0.02 | 0.78 | 0.01 | 0.12 | 0.01 | 0.01 | 0.01 | 0.00 | 0.00 | 0.00 |
| PERIOD 7 | -0.02 | 0.00 | 0.01 | 0.01 | 0.01 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| PERIOD 8 | 0.00 | 0.00 | 0.01 | 0.01 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| PERIOD 9 | 0.01 | -0.10 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| PERIOD 10 | 0.00 | -0.07 | 0.01 | 0.01 | 0.00 | -0.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| PERIOD 11 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| PERIOD 12 | 0.25 | 0.50 | 0.00 | 0.00 | 0.25 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| PERIOD 13 | -0.40 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| PERIOD 14 | 0.00 | 0.06 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| PERIOD 15 | -0.03 | -0.02 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| PERIOD 16 | 0.59 | -0.02 | -0.01 | 0.01 | -0.03 | -0.02 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 |
| PERIOD 17 | 0.01 | 0.07 | 0.06 | 0.00 | -0.34 | -0.02 | -0.02 | -0.02 | -0.01 | 0.00 | 0.00 |
| PERIOD 18 | 0.03 | 0.17 | -0.78 | 0.00 | -0.13 | -0.03 | -0.03 | -0.02 | -0.01 | 0.00 | 0.00 |
| PERIOD 19 | 0.04 | 0.00 | 0.00 | -0.03 | -0.04 | -0.04 | -0.02 | -0.01 | 0.00 | 0.00 | 0.00 |
| PERIOD 20 | 0.09 | 0.13 | 0.10 | -0.03 | -0.05 | 0.00 | 0.04 | 0.00 | 0.00 | 0.00 | 0.00 |
| PERIOD 21 | 0.01 | -0.02 | -0.05 | -0.03 | -0.01 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| PERIOD 22 | 0.02 | 0.03 | -0.01 | 0.12 | -0.02 | 0.00 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 |
| PERIOD 23 | -0.02 | 0.05 | 0.03 | 0.00 | 0.00 | 0.08 | 0.02 | 0.08 | 0.00 | 0.00 | 0.00 |
| PERIOD 24 | -0.22 | -0.36 | 0.03 | 0.00 | -0.22 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| PERIOD 25 | 0.05 | 0.00 | 0.05 | 0.00 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| PERIOD 26 | -0.06 | -0.08 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| PERIOD 27 | -0.03 | -0.04 | -0.03 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| PERIOD 28 | -0.47 | 0.34 | -0.01 | -0.06 | -0.03 | -0.04 | -0.02 | -0.01 | 0.00 | 0.00 | 0.00 |
| PERIOD 29 | 0.03 | -0.03 | -0.04 | 0.01 | 0.00 | -0.02 | -0.03 | -0.03 | -0.03 | -0.02 | -0.01 |
| PERIOD 30 | 0.38 | -0.09 | 0.03 | 0.18 | 0.00 | -0.02 | -0.03 | -0.05 | -0.05 | -0.04 | -0.03 |
| PERIOD 31 | 0.09 | 0.17 | 0.06 | 0.00 | -0.02 | -0.05 | -0.05 | -0.04 | -0.02 | 0.00 | 0.00 |
| PERIOD 32 | -0.05 | 0.11 | -0.09 | 0.06 | 0.07 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| PERIOD 33 | 0.02 | -0.03 | -0.05 | -0.08 | -0.07 | -0.08 | -0.02 | 0.00 | 0.00 | 0.00 | 0.00 |

Table 61. $NHAS_{2,j,1}$ continued

| | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| PERIOD 34 | 0.06 | -0.05 | -0.04 | -0.20 | -0.05 | -0.06 | -0.07 | -0.04 | -0.02 | -0.01 | 0.00 |
| PERIOD 35 | 0.03 | -0.15 | -0.12 | 0.04 | -0.04 | -0.09 | -0.02 | -0.08 | 0.00 | 0.00 | 0.00 |
| PERIOD 36 | -0.08 | -0.15 | -0.03 | 0.00 | -0.03 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| PERIOD 37 | -0.08 | -0.01 | -0.05 | 0.01 | -0.02 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| PERIOD 38 | -0.04 | -0.04 | -0.02 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| PERIOD 39 | -0.02 | -0.05 | -0.05 | 0.29 | -0.03 | 0.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| PERIOD 40 | 0.01 | -0.05 | 0.02 | 0.35 | 0.01 | -0.03 | -0.01 | -0.05 | -0.04 | -0.02 | -0.01 |
| PERIOD 41 | 0.08 | 0.03 | 0.02 | 0.48 | 0.15 | 0.19 | 0.05 | 0.02 | -0.04 | -0.04 | -0.05 |
| PERIOD 42 | -0.31 | 0.30 | 0.03 | 0.02 | 0.13 | 0.01 | -0.01 | 0.06 | -0.02 | -0.02 | -0.02 |
| PERIOD 43 | -0.03 | 0.04 | -0.04 | 0.53 | 0.02 | 0.01 | 0.12 | 0.05 | -0.05 | -0.04 | -0.02 |
| PERIOD 44 | 0.19 | -0.01 | -0.03 | 0.19 | 0.07 | -0.12 | -0.12 | -0.06 | -0.03 | 0.02 | 0.00 |
| PERIOD 45 | 0.12 | 0.17 | 0.02 | 0.06 | 0.02 | -0.02 | -0.03 | 0.02 | 0.01 | -0.06 | 0.00 |
| PERIOD 46 | 0.17 | 0.08 | -0.01 | 0.00 | 0.03 | 0.12 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 |
| PERIOD 47 | -0.13 | -0.02 | 0.01 | -0.04 | 0.03 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| PERIOD 48 | -0.02 | -0.01 | 0.00 | 0.00 | 0.00 | -0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| PERIOD 49 | -0.04 | -0.01 | -0.01 | -0.01 | 0.00 | -0.01 | 0.00 | 0.05 | 0.00 | 0.00 | 0.00 |
| PERIOD 50 | 0.00 | 0.00 | -0.01 | -0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| PERIOD 51 | 0.00 | 0.07 | 0.00 | -0.34 | 0.02 | 0.07 | 0.00 | 0.06 | 0.00 | 0.00 | 0.00 |
| PERIOD 52 | -0.06 | -0.24 | 0.00 | -0.34 | -0.02 | 0.12 | 0.03 | 0.10 | 0.00 | 0.00 | 0.00 |
| PERIOD 53 | -0.02 | 0.08 | 0.02 | -0.38 | -0.14 | -0.15 | 0.28 | 0.07 | 0.01 | 0.00 | 0.01 |
| PERIOD 54 | -0.01 | -0.35 | -0.02 | -0.06 | -0.13 | 0.03 | 0.47 | 0.00 | 0.01 | 0.01 | 0.02 |
| PERIOD 55 | 0.04 | -0.11 | 0.03 | -0.47 | 0.01 | 0.13 | -0.04 | 0.14 | 0.01 | 0.02 | 0.01 |
| PERIOD 56 | -0.18 | -0.22 | -0.01 | -0.28 | -0.17 | -0.03 | 0.01 | 0.22 | 0.02 | -0.02 | 0.00 |
| PERIOD 57 | -0.14 | -0.18 | -0.01 | -0.06 | -0.04 | 0.00 | 0.01 | 0.19 | -0.01 | 0.06 | 0.00 |
| PERIOD 58 | -0.24 | -0.07 | 0.00 | 0.00 | -0.04 | -0.12 | 0.01 | 0.03 | 0.00 | 0.00 | 0.00 |
| PERIOD 59 | 0.01 | 0.01 | -0.01 | -0.07 | -0.04 | -0.02 | -0.01 | 0.02 | 0.00 | 0.00 | 0.00 |
| PERIOD 60 | 0.01 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| PERIOD 61 | 0.01 | 0.03 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | -0.05 | 0.00 | 0.00 | 0.00 |
| PERIOD 62 | 0.00 | 0.05 | 0.01 | 0.00 | 0.00 | 0.00 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 |
| PERIOD 63 | 0.01 | -0.06 | 0.02 | 0.17 | 0.00 | -0.20 | 0.00 | 0.04 | 0.00 | 0.00 | 0.00 |
| PERIOD 64 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | -0.12 | -0.06 | 0.20 | 0.01 | 0.01 | 0.00 |
| PERIOD 65 | -0.04 | -0.09 | -0.02 | -0.09 | 0.00 | -0.03 | -0.34 | 0.16 | 0.00 | 0.01 | 0.01 |
| PERIOD 66 | -0.01 | -0.02 | 0.01 | -0.07 | 0.00 | -0.04 | -0.47 | 0.07 | 0.00 | 0.01 | 0.01 |
| PERIOD 67 | -0.07 | -0.01 | -0.05 | 0.07 | -0.01 | -0.14 | -0.09 | -0.10 | 0.02 | 0.02 | 0.01 |

## Appendix I. *Autocorrelations of NHAS for Event Type 1 and Time Block 1*

This appendix contains the autocorrelations of $NHAS_{ijk}$ for event type 2 and time block 1. The autocorrelations were caluclated for the first 17 lags for all twenty-two geographical regions. Each column represents a geographical region and each row represents a lag. Any autocorrelation that is greater than 0.24 or less than -0.24 is significantly different from zero.

Table 62. Autocorrelations of $NHAS_{2,j,1}$

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| LAG 1 | 0.21 | 0.14 | 0.00 | -0.02 | -0.05 | 0.29 | 0.31 | 0.04 | 0.08 | 0.16 | 0.50 |
| LAG 2 | 0.00 | -0.01 | 0.01 | -0.18 | -0.22 | 0.11 | 0.22 | -0.01 | 0.15 | 0.23 | 0.35 |
| LAG 3 | 0.02 | -0.13 | 0.00 | 0.32 | 0.03 | 0.07 | 0.23 | 0.12 | 0.08 | 0.02 | 0.15 |
| LAG 4 | -0.03 | -0.02 | -0.10 | -0.04 | -0.01 | 0.10 | -0.01 | 0.13 | -0.04 | 0.13 | 0.01 |
| LAG 5 | -0.02 | -0.02 | 0.00 | 0.04 | 0.01 | -0.02 | 0.05 | -0.01 | 0.04 | -0.09 | 0.06 |
| LAG 6 | -0.02 | -0.02 | 0.00 | -0.07 | 0.00 | 0.05 | -0.02 | -0.03 | -0.24 | -0.05 | 0.08 |
| LAG 7 | -0.03 | -0.02 | 0.00 | -0.02 | -0.01 | 0.09 | -0.12 | 0.14 | 0.14 | -0.07 | 0.15 |
| LAG 8 | -0.01 | -0.03 | 0.19 | 0.20 | 0.00 | 0.21 | -0.15 | -0.07 | -0.05 | -0.17 | 0.24 |
| LAG 9 | -0.02 | -0.04 | 0.00 | -0.04 | -0.03 | 0.07 | -0.24 | -0.08 | -0.09 | -0.13 | 0.19 |
| LAG 10 | -0.02 | -0.01 | 0.14 | -0.10 | -0.13 | 0.00 | -0.07 | -0.13 | -0.04 | -0.18 | 0.05 |
| LAG 11 | 0.01 | -0.05 | -0.12 | -0.05 | 0.14 | 0.03 | -0.03 | -0.06 | -0.10 | -0.10 | -0.09 |
| LAG 12 | -0.19 | -0.23 | -0.28 | -0.08 | -0.33 | -0.28 | -0.36 | -0.17 | -0.23 | -0.39 | -0.31 |
| LAG 13 | -0.04 | -0.13 | 0.00 | -0.04 | -0.07 | -0.13 | -0.03 | -0.08 | -0.11 | -0.07 | -0.17 |
| LAG 14 | -0.03 | -0.02 | -0.05 | 0.13 | 0.19 | -0.02 | -0.07 | -0.04 | -0.03 | 0.02 | -0.23 |
| LAG 15 | -0.05 | 0.09 | 0.00 | 0.03 | -0.02 | -0.08 | -0.20 | -0.04 | -0.05 | 0.17 | -0.25 |
| LAG 16 | -0.03 | -0.01 | 0.00 | -0.07 | 0.00 | -0.28 | -0.05 | 0.01 | -0.01 | 0.04 | -0.08 |
| LAG 17 | -0.03 | -0.01 | -0.14 | 0.09 | 0.00 | -0.06 | -0.18 | -0.05 | -0.11 | 0.23 | -0.18 |

Table 63. Autocorrelations of $NHAS_{2,j,1}$ continued

| | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| LAG 1 | -0.01 | 0.08 | -0.02 | 0.53 | 0.31 | 0.07 | 0.48 | 0.55 | 0.54 | 0.00 | 0.52 |
| LAG 2 | -0.09 | 0.11 | -0.04 | 0.49 | 0.15 | -0.06 | 0.10 | 0.16 | 0.23 | 0.34 | 0.25 |
| LAG 3 | -0.23 | 0.09 | 0.06 | 0.36 | 0.14 | -0.01 | -0.02 | -0.01 | 0.23 | 0.12 | 0.02 |
| LAG 4 | 0.18 | -0.06 | 0.01 | 0.32 | 0.07 | 0.12 | -0.03 | 0.11 | 0.06 | 0.14 | -0.02 |
| LAG 5 | 0.05 | 0.04 | -0.01 | 0.12 | -0.12 | 0.31 | 0.00 | 0.07 | -0.07 | 0.07 | -0.02 |
| LAG 6 | -0.09 | -0.03 | -0.01 | -0.01 | 0.04 | -0.03 | 0.00 | 0.13 | -0.02 | -0.01 | -0.02 |
| LAG 7 | -0.09 | -0.34 | -0.03 | -0.14 | 0.24 | -0.17 | -0.01 | 0.21 | -0.03 | 0.05 | -0.02 |
| LAG 8 | -0.11 | -0.02 | -0.01 | -0.18 | -0.11 | -0.20 | 0.01 | 0.28 | -0.03 | 0.02 | -0.02 |
| LAG 9 | 0.05 | -0.04 | -0.01 | -0.17 | -0.03 | 0.24 | -0.05 | 0.28 | 0.05 | 0.01 | -0.02 |
| LAG 10 | 0.04 | -0.04 | -0.01 | -0.27 | -0.05 | 0.11 | -0.05 | 0.10 | 0.13 | 0.04 | 0.02 |
| LAG 11 | 0.15 | 0.12 | 0.06 | -0.26 | -0.14 | -0.04 | -0.14 | -0.04 | 0.20 | 0.22 | 0.23 |
| LAG 12 | -0.46 | -0.30 | -0.51 | -0.49 | -0.47 | -0.42 | -0.45 | -0.02 | 0.10 | -0.22 | 0.05 |
| LAG 13 | -0.01 | -0.07 | -0.04 | -0.19 | -0.15 | -0.04 | -0.23 | 0.14 | 0.14 | 0.20 | -0.06 |
| LAG 14 | 0.30 | 0.11 | 0.11 | -0.21 | -0.09 | 0.24 | -0.06 | 0.12 | 0.10 | -0.23 | -0.11 |
| LAG 15 | 0.40 | -0.07 | -0.01 | -0.22 | -0.11 | -0.04 | 0.01 | 0.04 | -0.16 | 0.05 | -0.05 |
| LAG 16 | -0.07 | 0.11 | 0.02 | -0.21 | 0.00 | 0.01 | 0.01 | -0.04 | -0.15 | -0.08 | -0.03 |
| LAG 17 | -0.18 | 0.05 | 0.09 | -0.02 | -0.03 | -0.26 | 0.00 | -0.07 | -0.06 | -0.11 | -0.03 |

## Appendix J. *Autocorrelations for Each of the Twenty-Two Geographical Regions*

This appendix contains twenty-two two-dimensional plots of the autocorrelations of $NHAS_{2,j,1}$ for each of the twenty-two geographical regions. The autocorrelatoins were calculated for the first seventeen lags. Any autocorrelation that is gretaer than 0.24 or less than -0.24 is significantly different from zero.

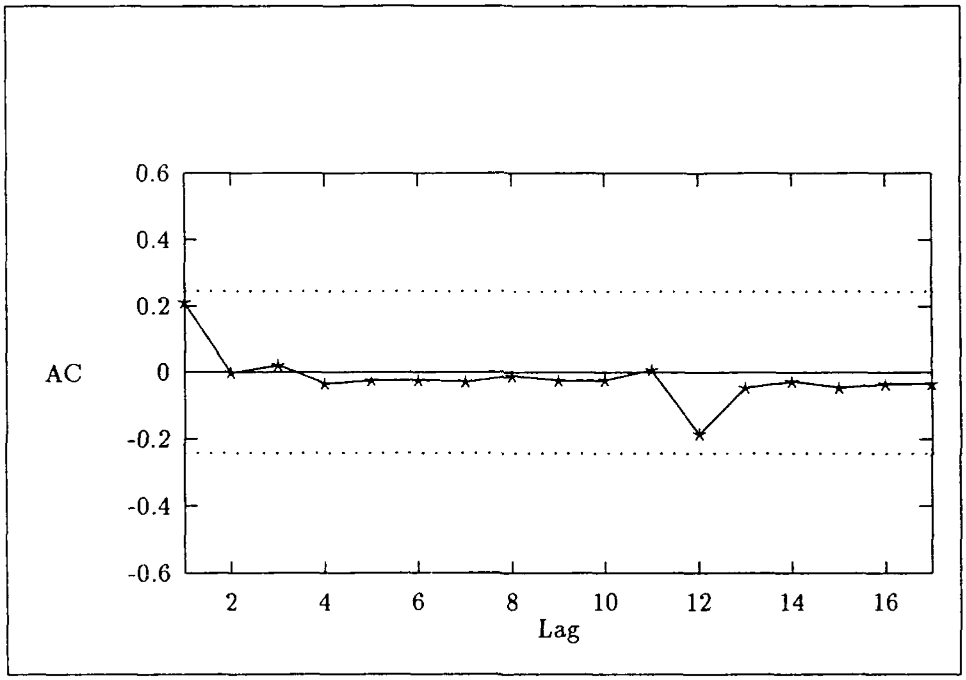


Figure 51. Autocorrelations for Region 1 $NHAS_{2,1,1}$

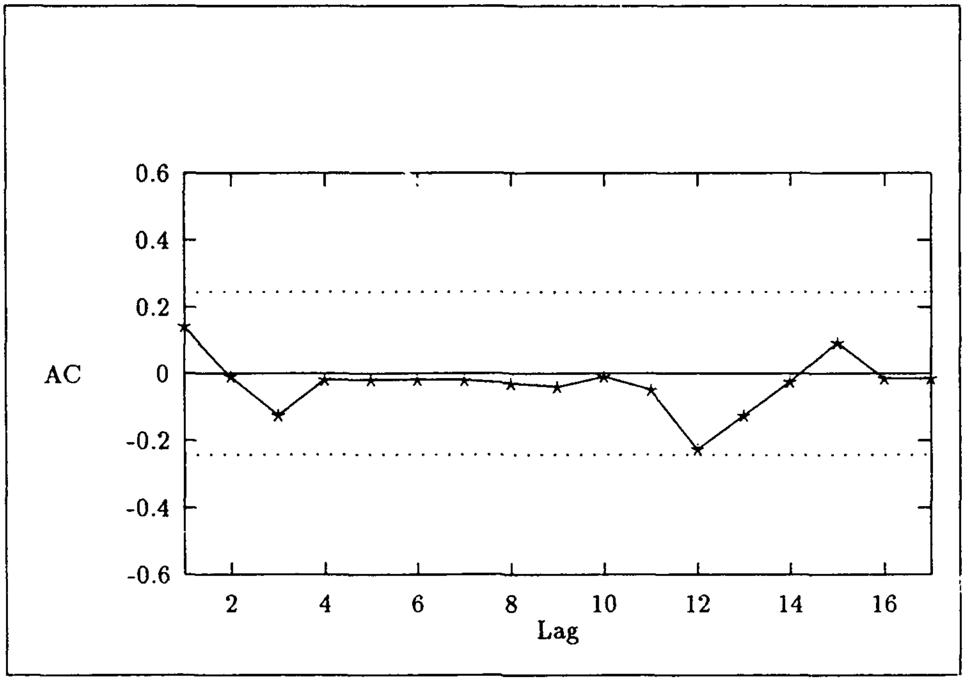


Figure 52. Autocorrelations for Region 2 $NHAS_{2,2,1}$

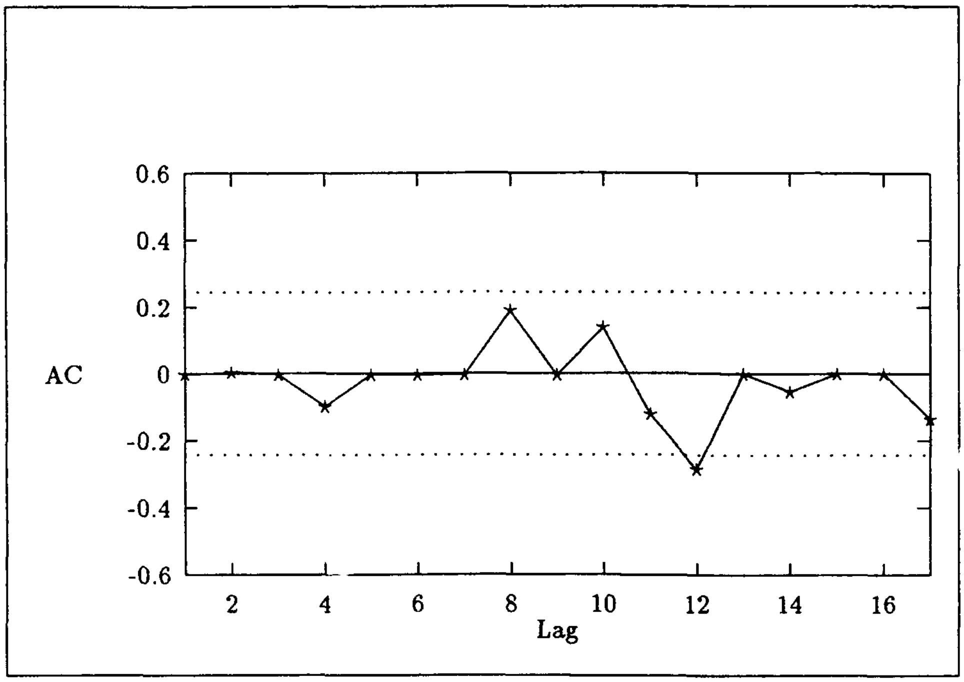


Figure 53. Autocorrelations for Region 3 $NHAS_{2,3,1}$

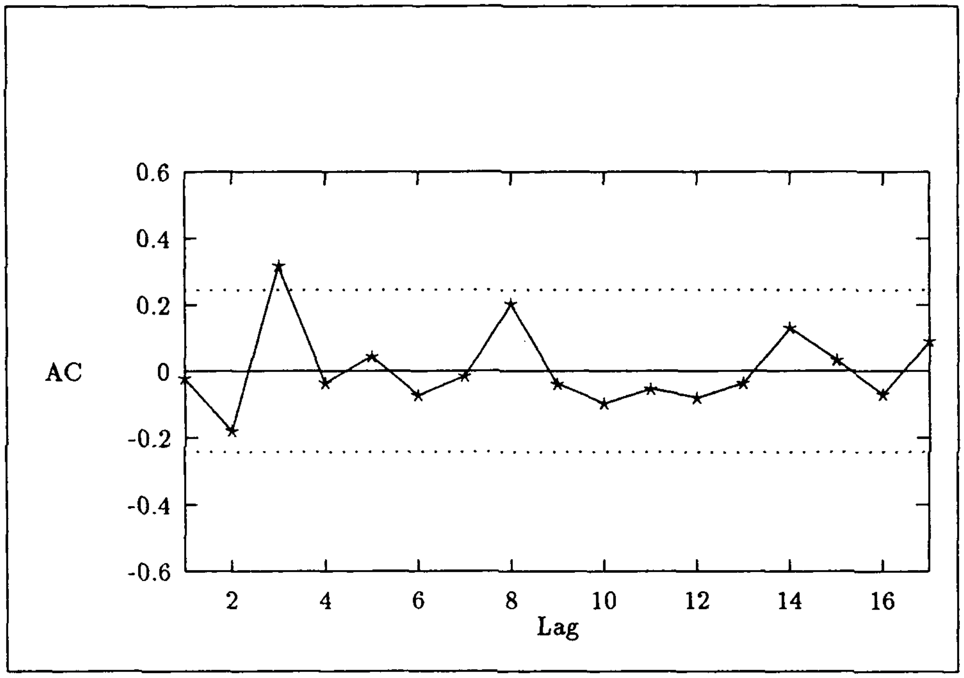


Figure 54. Autocorrelations for Region 4 $NHAS_{2,4,1}$

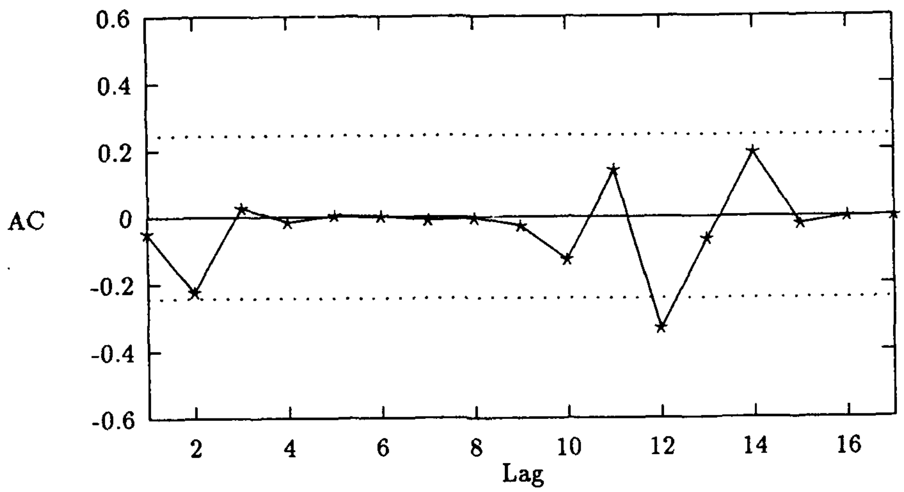


Figure 55. Autocorrelations for Region 5 $NHAS_{2,5,1}$

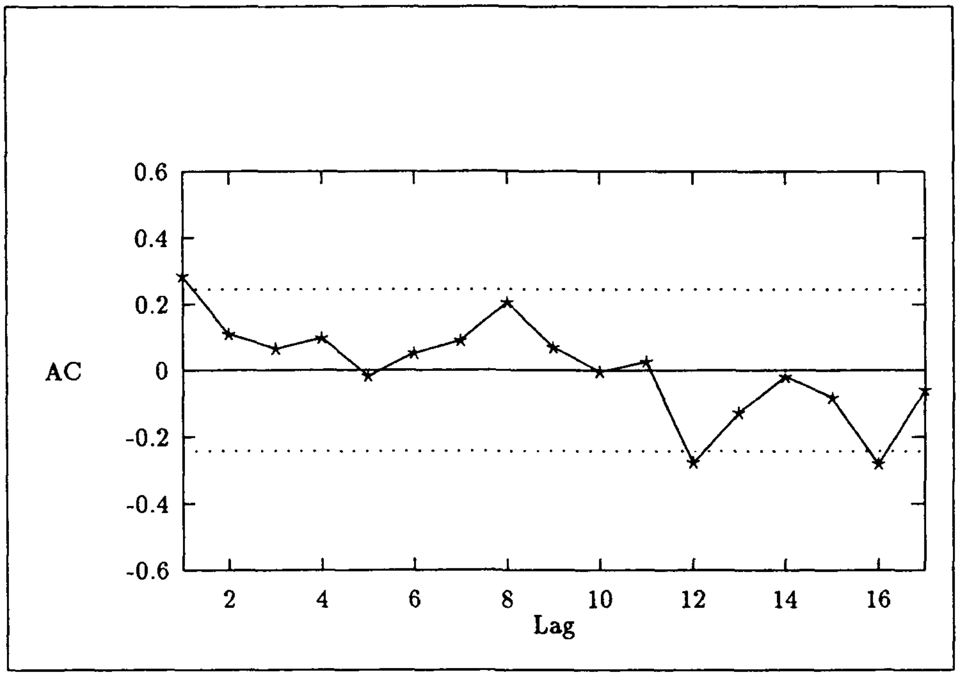


Figure 56. Autocorrelations for Region 6 $NHAS_{2,6,1}$

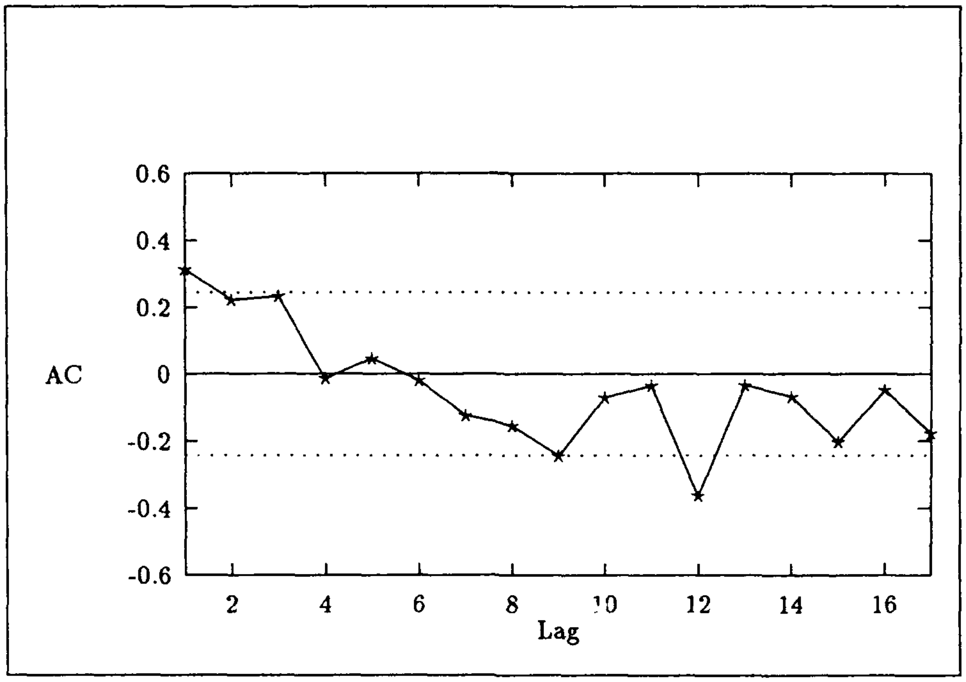


Figure 57. Autocorrelations for Region 7 $NHAS_{2,7,1}$

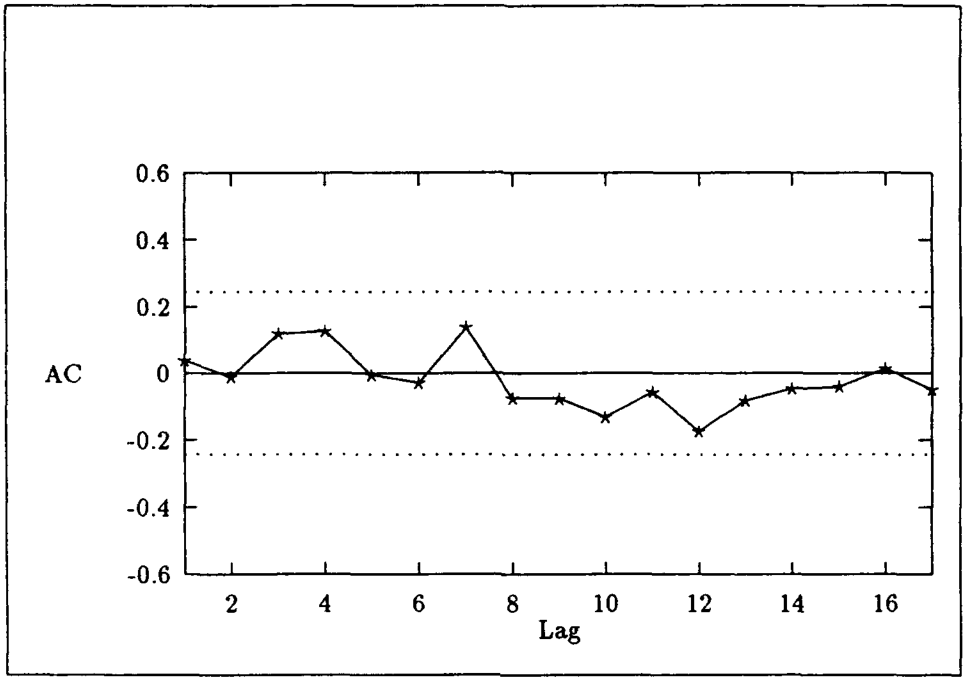


Figure 58. Autocorrelations for Region 8 $NHAS_{2,8,1}$

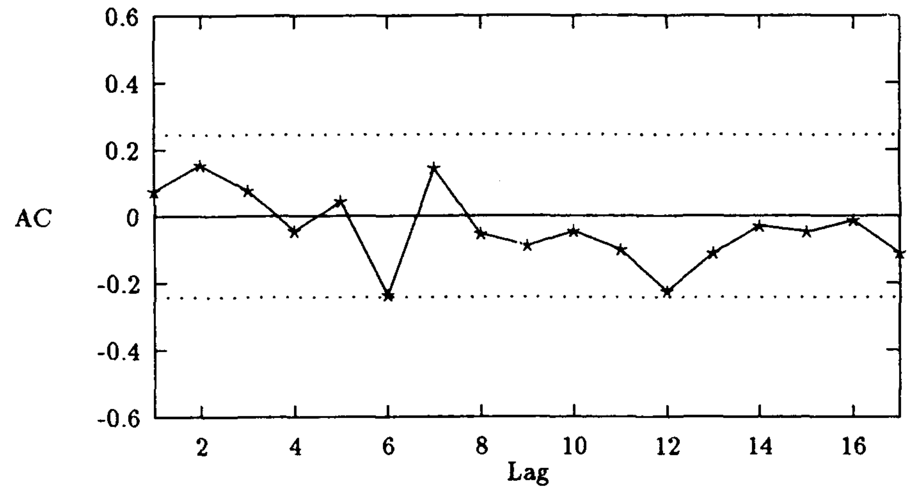


Figure 59. Autocorrelations for Region 9 $NHAS_{2,9,1}$

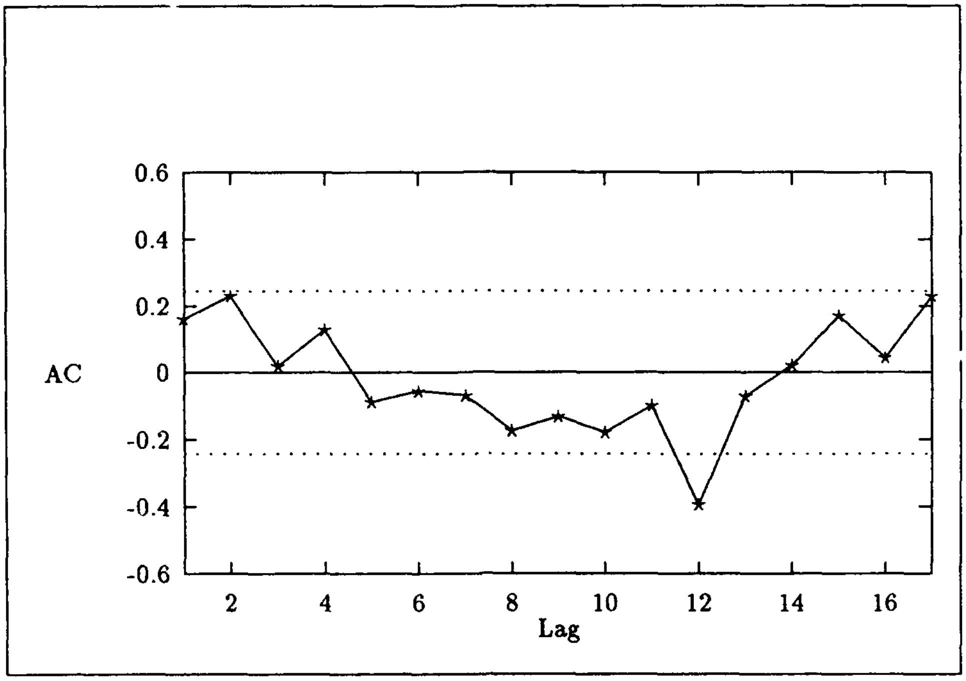


Figure 60. Autocorrelations for Region 10 $NHAS_{2,10,1}$

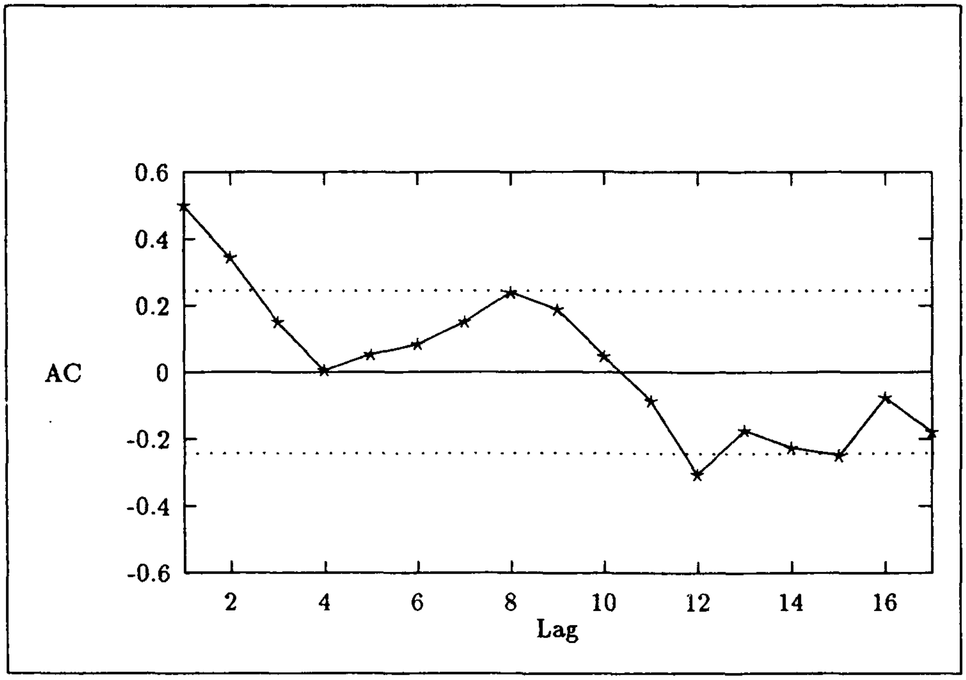


Figure 61. Autocorrelations for Region 11 $NHAS_{\cdot,11,1}$

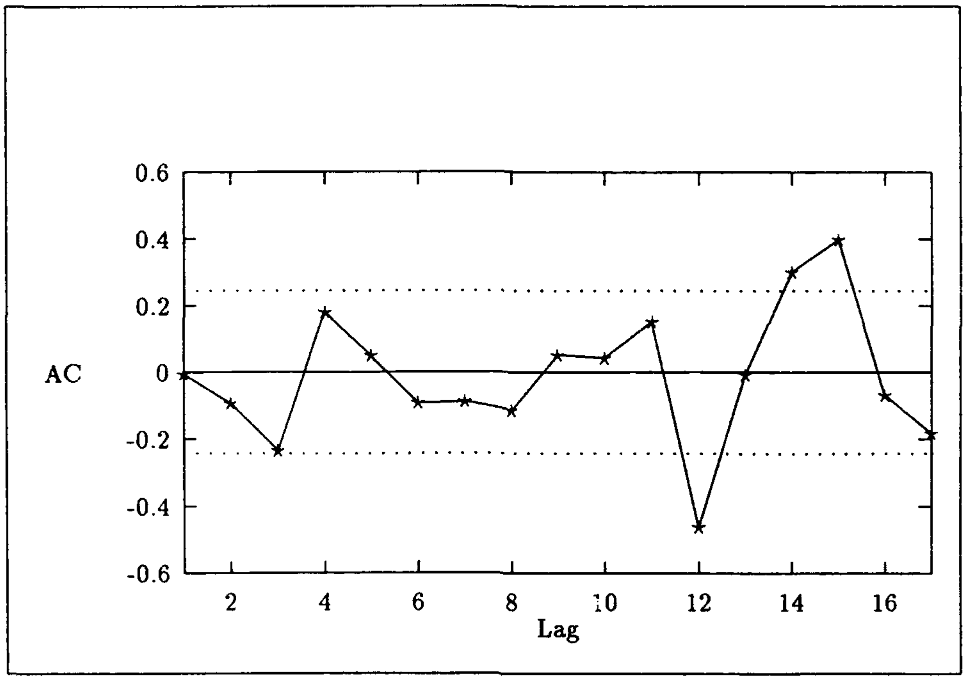


Figure 62. Autocorrelations for Region 12 $NHAS_{2,12,1}$

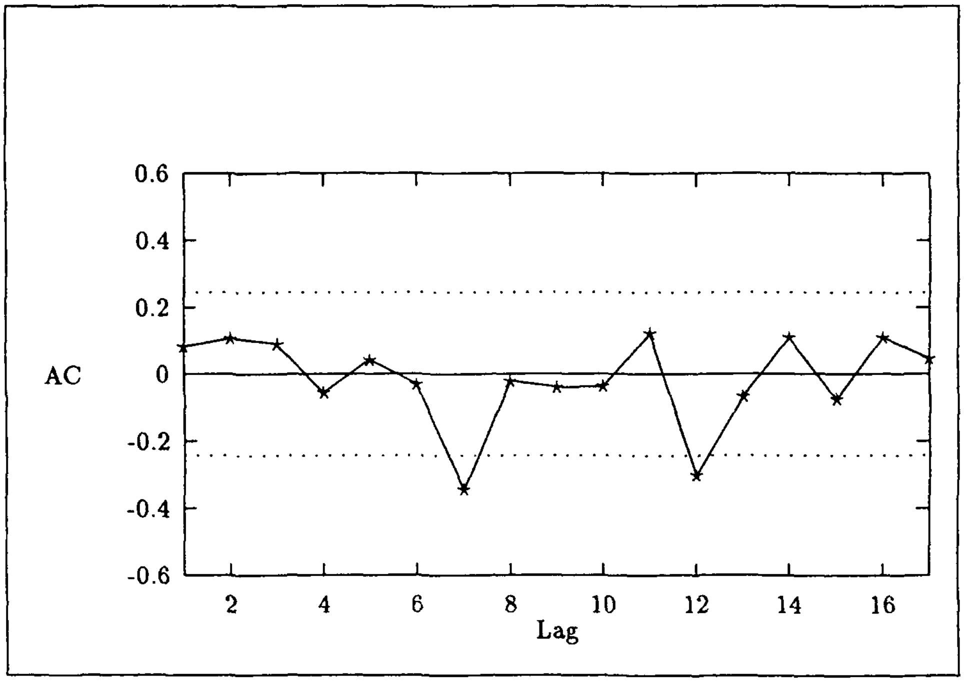


Figure 63. Autocorrelations for Region 13 $NHAS_{2,13,1}$

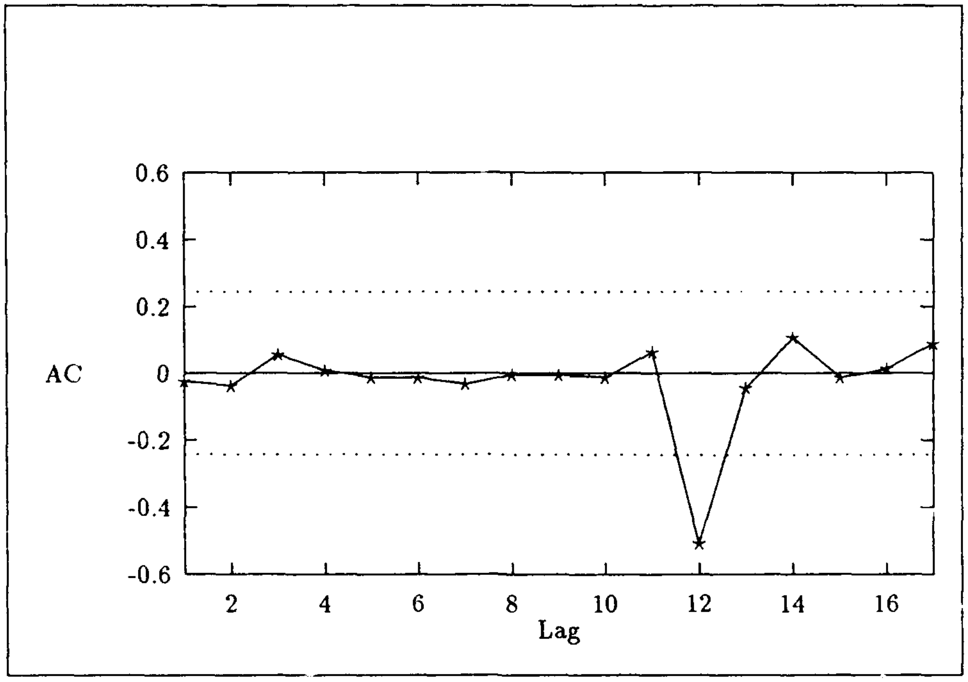


Figure 64. Autocorrelations for Region 14 $NHAS_{2,14,1}$

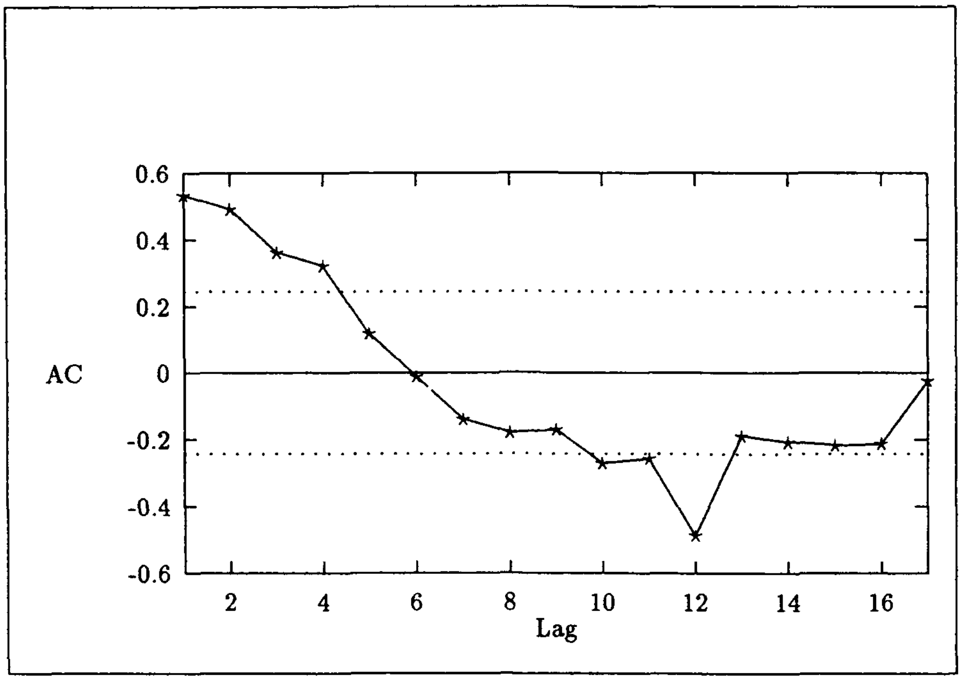


Figure 65. Autocorrelations for Region 15 $NHAS_{2,15,1}$

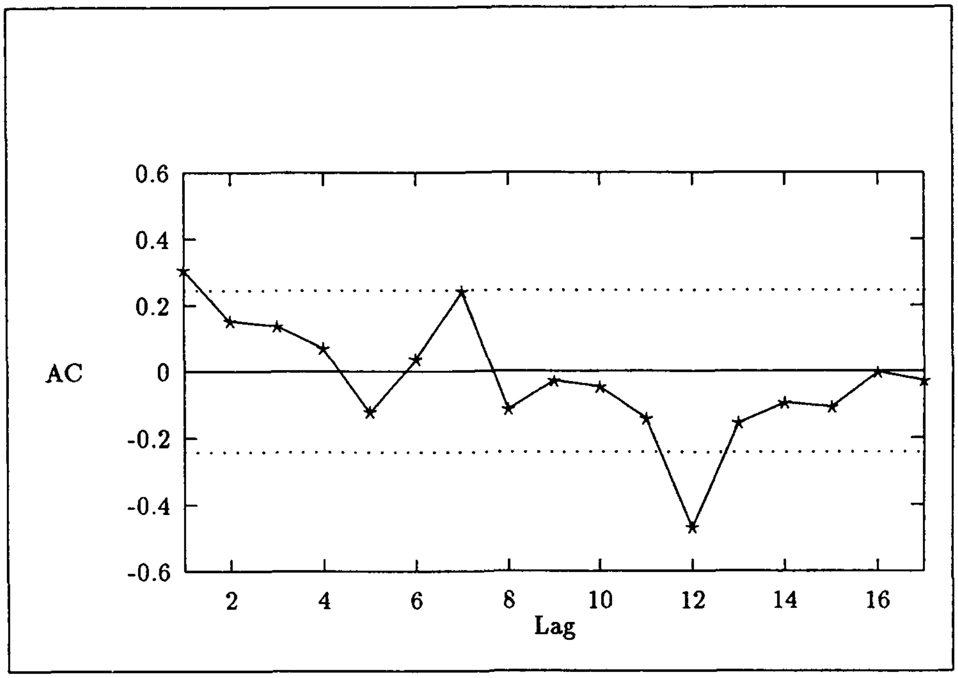


Figure 66. Autocorrelations for Region 16 $NHAS_{2,16,1}$

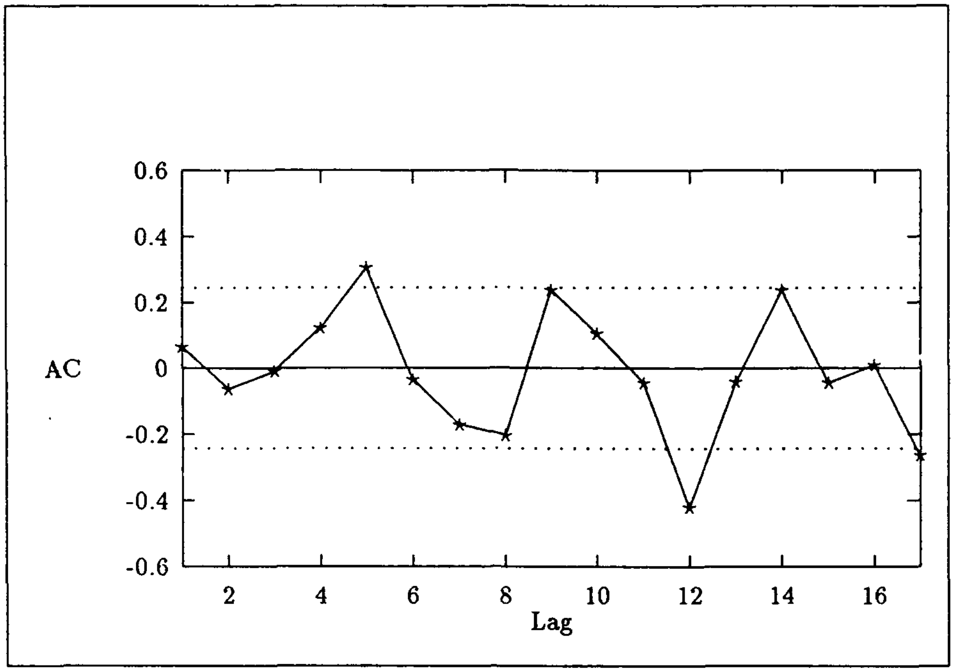


Figure 67. Autocorrelations for Region 17 $NHAS_{2,17,1}$

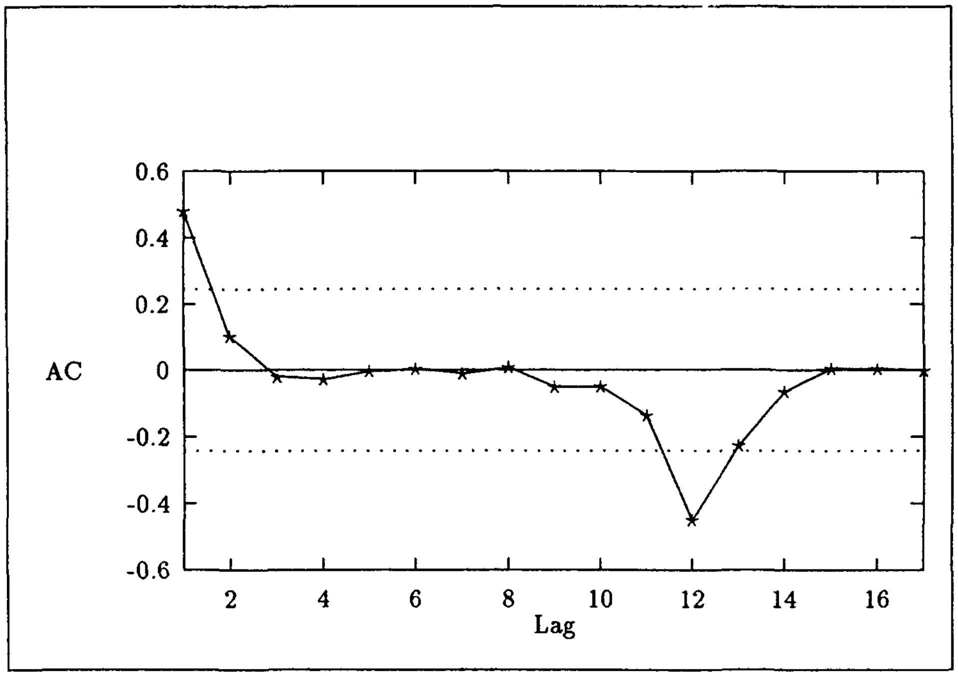


Figure 68. Autocorrelations for Region 18 $NHAS_{2,18,1}$

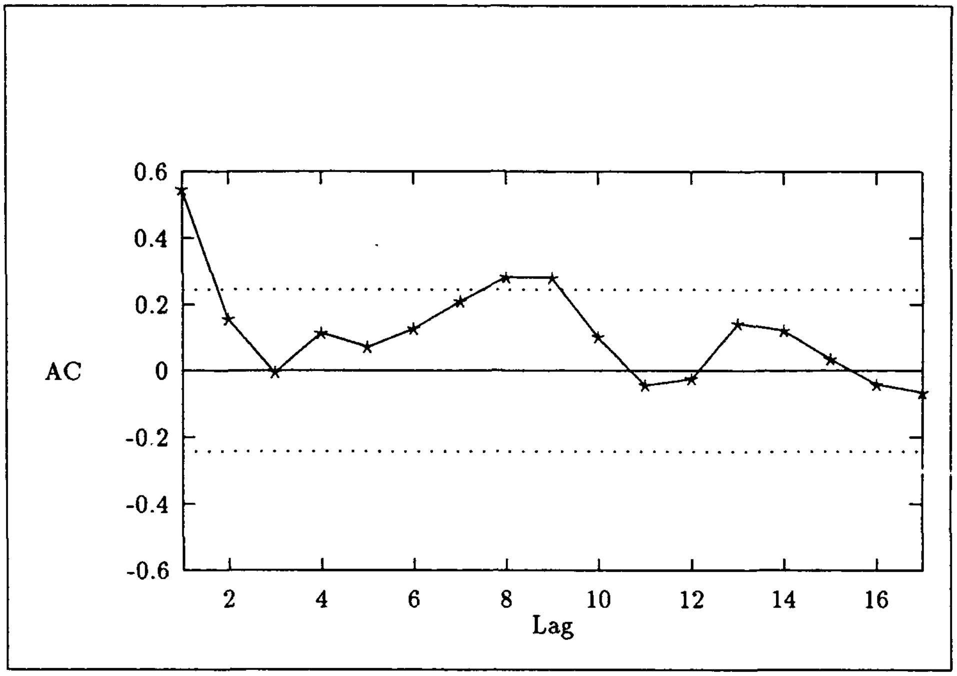


Figure 69. Autocorrelations for Region 19 $NHAS_{2,19,1}$

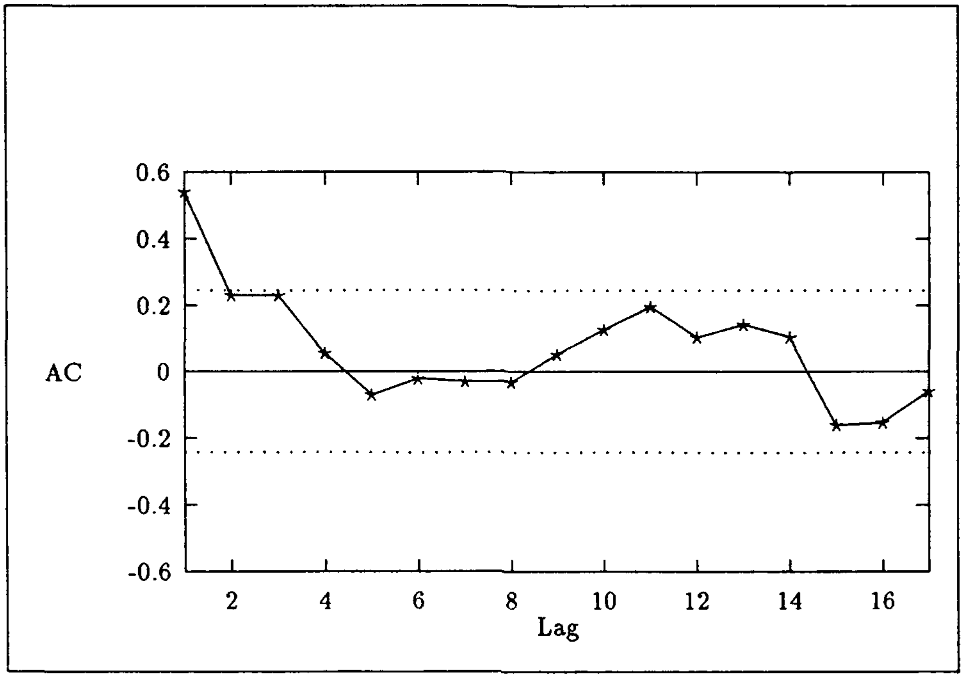


Figure 70. Autocorrelations for Region 20 $NHAS_{2,20,1}$

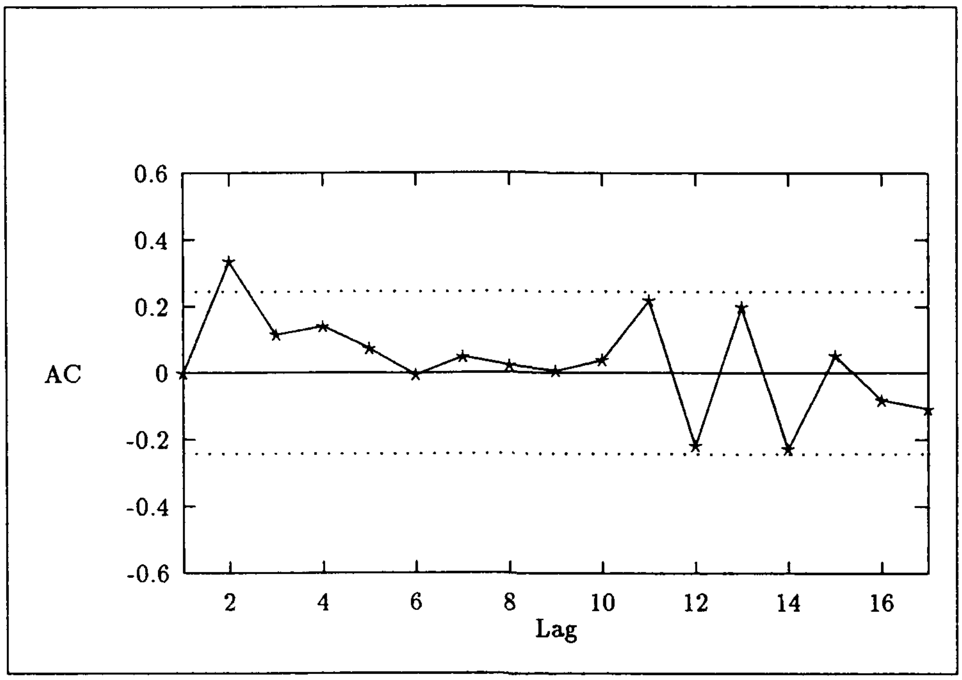


Figure 71. Autocorrelations for Region 21 $NHAS_{2,21,1}$

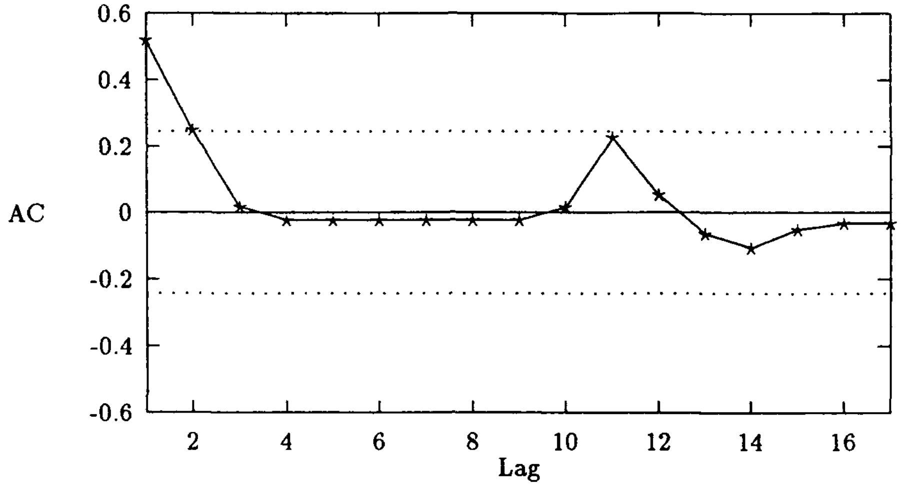


Figure 72. Autocorrelations for Region 22 $NHAS_{2,22,1}$

# Appendix K. *Partial Autocorrelatoins of Target Region 11 NHAS for Event Type 2 and Time Block 1*

This appendix contains the partial autocorrelations of NHAS$X_{ijk}$ for event type 2, geographical region 11, and time block 1. The partial autocorrelations were calculated for the first seventeen lags. Any partial autocorrelation that is greater than 0.24 or less than -0.24 is significantly different from zero. Each row represents a lag.

Table 64. Partial Autocorrelations of $NHAS_{2,11,1}$

| | PAC |
|---|---|
| LAG 1 | 0.50 |
| LAG 2 | 0.13 |
| LAG 3 | -0.08 |
| LAG 4 | -0.10 |
| LAG 5 | 0.12 |
| LAG 6 | 0.08 |
| LAG 7 | 0.08 |
| LAG 8 | 0.13 |
| LAG 9 | -0.01 |
| LAG 10 | -0.15 |
| LAG 11 | -0.13 |
| LAG 12 | -0.26 |
| LAG 13 | 0.14 |
| LAG 14 | -0.13 |
| LAG 15 | -0.22 |
| LAG 16 | 0.12 |
| LAG 17 | -0.15 |

# Appendix L. *Combined Series With Target Region 11*

This appendix contains the the $NHAS_{2,j,1}$ values for target region 11, the weighted sum of the first order neighbors of target region 11, and the weighted sum of the second order neighbors of target region 11. The combined series results if the rows are read across. Each row represents a period.

Table 65. Combined Series for Target Region 11

| | TARGET 11 | 1ST ORDER | 2ND ORDER |
|---|---|---|---|
| PERIOD 1 | 0.27 | -0.04 | -0.11 |
| PERIOD 2 | 0.00 | 0.02 | -0.10 |
| PERIOD 3 | 0.01 | -0.15 | 0.11 |
| PERIOD 4 | -0.01 | -0.16 | -0.05 |
| PERIOD 5 | -0.01 | -0.07 | -0.06 |
| PERIOD 6 | 0.00 | -0.10 | 0.10 |
| PERIOD 7 | -0.01 | 0.00 | 0.00 |
| PERIOD 8 | -0.01 | 0.00 | 0.00 |
| PERIOD 9 | 0.02 | -0.02 | -0.16 |
| PERIOD 10 | 0.00 | -0.14 | -0.07 |
| PERIOD 11 | -0.33 | -0.13 | 0.00 |
| PERIOD 12 | 0.00 | -0.01 | -0.08 |
| PERIOD 13 | -0.40 | -0.01 | -0.03 |
| PERIOD 14 | 0.00 | 0.01 | 0.11 |
| PERIOD 15 | -0.01 | 0.11 | -0.08 |
| PERIOD 16 | 0.02 | 0.04 | 0.12 |
| PERIOD 17 | 0.01 | 0.16 | 0.00 |
| PERIOD 18 | 0.02 | 0.05 | -0.06 |
| PERIOD 19 | 0.03 | -0.01 | 0.00 |
| PERIOD 20 | 0.07 | 0.20 | 0.05 |
| PERIOD 21 | 0.10 | 0.26 | 0.07 |
| PERIOD 22 | 0.31 | 0.08 | 0.06 |
| PERIOD 23 | -0.05 | 0.28 | 0.07 |
| PERIOD 24 | 0.15 | 0.20 | -0.01 |
| PERIOD 25 | 0.03 | 0.28 | 0.08 |
| PERIOD 26 | -0.10 | 0.19 | -0.06 |
| PERIOD 27 | -0.04 | 0.20 | -0.08 |
| PERIOD 28 | 0.10 | 0.03 | -0.04 |
| PERIOD 29 | -0.01 | -0.10 | -0.04 |
| PERIOD 30 | 0.52 | 0.03 | -0.07 |
| PERIOD 31 | 0.62 | 0.06 | 0.04 |
| PERIOD 32 | 0.11 | -0.07 | -0.02 |
| PERIOD 33 | 0.09 | -0.07 | 0.08 |

Table 66. Combined Series for Target Region 11 continued

| | TARGET 11 | 1ST ORDER | 2ND ORDER |
|---|---|---|---|
| PERIOD 34 | -0.23 | 0.15 | 0.09 |
| PERIOD 35 | -0.03 | -0.01 | 0.04 |
| PERIOD 36 | -0.12 | -0.04 | 0.07 |
| PERIOD 37 | -0.13 | -0.06 | 0.06 |
| PERIOD 38 | 0.31 | -0.10 | 0.03 |
| PERIOD 39 | 0.01 | -0.07 | 0.03 |
| PERIOD 40 | -0.04 | 0.12 | -0.04 |
| PERIOD 41 | 0.03 | 0.11 | 0.04 |
| PERIOD 42 | -0.46 | 0.14 | -0.01 |
| PERIOD 43 | -0.55 | 0.19 | 0.00 |
| PERIOD 44 | -0.02 | -0.04 | 0.03 |
| PERIOD 45 | -0.12 | 0.07 | -0.06 |
| PERIOD 46 | 0.00 | -0.07 | 0.00 |
| PERIOD 47 | -0.04 | 0.29 | -0.05 |
| PERIOD 48 | -0.16 | 0.09 | 0.01 |
| PERIOD 49 | -0.04 | 0.39 | -0.07 |
| PERIOD 50 | -0.33 | 0.07 | 0.04 |
| PERIOD 51 | 0.02 | -0.10 | 0.06 |
| PERIOD 52 | -0.02 | -0.06 | 0.08 |
| PERIOD 53 | 0.22 | -0.09 | 0.03 |
| PERIOD 54 | 0.21 | -0.13 | 0.04 |
| PERIOD 55 | 0.24 | -0.15 | 0.00 |
| PERIOD 56 | 0.45 | 0.04 | -0.02 |
| PERIOD 57 | 0.71 | -0.18 | -0.04 |
| PERIOD 58 | 0.50 | 0.00 | -0.09 |
| PERIOD 59 | -0.01 | 0.17 | 0.00 |
| PERIOD 60 | 0.01 | 0.34 | -0.08 |
| PERIOD 61 | 0.02 | 0.16 | -0.03 |
| PERIOD 62 | 0.38 | 0.02 | -0.10 |
| PERIOD 63 | 0.07 | 0.09 | -0.05 |
| PERIOD 64 | 0.38 | 0.02 | -0.10 |
| PERIOD 65 | 0.34 | -0.02 | -0.05 |
| PERIOD 66 | 0.42 | -0.02 | -0.05 |
| PERIOD 67 | 0.37 | -0.01 | -0.03 |

# Appendix M. *Autocorrelations and Partial Autocorrelations of the Combined Series with Target Region 11*

This appendix contains the autocorrelations and the partial autocorrelations of the combined series with target region 11 for event type 2 and time block 1. The autocorrelations and the partial autocorrelations were calculated for the first forty-four lags. Any autocorrelations or partial autocorrelation that is greater than 0.14 or less than -0.14 is significantly different from zero. Each row represents a lag.

Table 67. Autocorrelations and Partial Autocorrelations for the Combined Series with Target Region 11

| | AC | PAC |
|---|---|---|
| LAG 1 | -0.13 | -0.13 |
| LAG 2 | -0.11 | -0.13 |
| LAG 3 | 0.49 | 0.47 |
| LAG 4 | -0.04 | 0.07 |
| LAG 5 | -0.08 | 0.00 |
| LAG 6 | 0.35 | 0.16 |
| LAG 7 | -0.01 | 0.04 |
| LAG 8 | -0.05 | 0.03 |
| LAG 9 | 0.17 | -0.06 |
| LAG 10 | 0.06 | 0.07 |
| LAG 11 | 0.07 | 0.16 |
| LAG 12 | 0.04 | -0.04 |
| LAG 13 | 0.01 | -0.07 |
| LAG 14 | 0.07 | -0.03 |
| LAG 15 | 0.04 | 0.05 |
| LAG 16 | 0.00 | 0.00 |
| LAG 17 | 0.10 | 0.02 |
| LAG 18 | 0.06 | 0.09 |
| LAG 19 | -0.07 | -0.04 |
| LAG 20 | 0.11 | 0.03 |
| LAG 21 | 0.12 | 0.06 |
| LAG 22 | -0.06 | 0.02 |

Table 68. Autocorrelations and Partial Autocorrelations for the Combined Series with Target Region 11 continued

| | AC | PAC |
|---|---|---|
| LAG 23 | 0.07 | 0.01 |
| LAG 24 | 0.16 | 0.07 |
| LAG 25 | -0.07 | 0.00 |
| LAG 26 | 0.01 | -0.06 |
| LAG 27 | 0.13 | -0.04 |
| LAG 28 | -0.01 | 0.04 |
| LAG 29 | 0.01 | 0.05 |
| LAG 30 | 0.01 | -0.12 |
| LAG 31 | -0.03 | -0.16 |
| LAG 32 | -0.04 | -0.10 |
| LAG 33 | -0.07 | -0.11 |
| LAG 34 | -0.01 | -0.06 |
| LAG 35 | -0.05 | -0.09 |
| LAG 36 | -0.27 | -0.27 |
| LAG 37 | -0.01 | -0.10 |
| LAG 38 | 0.01 | -0.02 |
| LAG 39 | -0.14 | 0.11 |
| LAG 40 | -0.05 | -0.03 |
| LAG 41 | 0.05 | 0.06 |
| LAG 42 | -0.19 | -0.06 |
| LAG 43 | -0.03 | 0.04 |
| LAG 44 | 0.06 | 0.01 |

## Appendix N. *Residuals from Fitting the* $SSTMA(2_{1,1})_{12}$ *Model*

This appendix contains the residuals from fitting the $\mathrm{SSTMA}(2_{1,1})_{12}$ model to the combined series with target region 11 for all 201 periods.

Table 69. Combined Series with Target Region 11 Residuals

| PERIOD | RESID | PERIOD | RESID | PERIOD | RESID | PERIOD | RESID |
|---|---|---|---|---|---|---|---|
| PERIOD 1 | 0.18 | PERIOD 26 | -0.06 | PERIOD 51 | -0.04 | PERIOD 76 | -0.18 |
| PERIOD 2 | -0.02 | PERIOD 27 | -0.13 | PERIOD 52 | 0.00 | PERIOD 77 | 0.01 |
| PERIOD 3 | -0.08 | PERIOD 28 | -0.03 | PERIOD 53 | -0.04 | PERIOD 78 | -0.03 |
| PERIOD 4 | -0.08 | PERIOD 29 | -0.12 | PERIOD 54 | -0.03 | PERIOD 79 | 0.02 |
| PERIOD 5 | 0.04 | PERIOD 30 | -0.06 | PERIOD 55 | -0.02 | PERIOD 80 | 0.10 |
| PERIOD 6 | -0.01 | PERIOD 31 | -0.22 | PERIOD 56 | -0.07 | PERIOD 81 | -0.10 |
| PERIOD 7 | -0.08 | PERIOD 32 | -0.18 | PERIOD 57 | -0.02 | PERIOD 82 | 0.09 |
| PERIOD 8 | -0.16 | PERIOD 33 | 0.07 | PERIOD 58 | 0.03 | PERIOD 83 | -0.07 |
| PERIOD 9 | 0.17 | PERIOD 34 | 0.09 | PERIOD 59 | 0.21 | PERIOD 84 | 0.04 |
| PERIOD 10 | 0.02 | PERIOD 35 | 0.03 | PERIOD 60 | 0.05 | PERIOD 85 | -0.12 |
| PERIOD 11 | -0.08 | PERIOD 36 | -0.06 | PERIOD 61 | 0.09 | PERIOD 86 | -0.10 |
| PERIOD 12 | -0.11 | PERIOD 37 | -0.25 | PERIOD 62 | 0.16 | PERIOD 87 | -0.09 |
| PERIOD 13 | -0.04 | PERIOD 38 | -0.06 | PERIOD 63 | -0.02 | PERIOD 88 | 0.47 |
| PERIOD 14 | 0.02 | PERIOD 39 | -0.07 | PERIOD 64 | 0.24 | PERIOD 89 | 0.11 |
| PERIOD 15 | -0.05 | PERIOD 40 | 0.00 | PERIOD 65 | -0.12 | PERIOD 90 | -0.09 |
| PERIOD 16 | -0.08 | PERIOD 41 | -0.03 | PERIOD 66 | -0.16 | PERIOD 91 | 0.40 |
| PERIOD 17 | -0.09 | PERIOD 42 | 0.11 | PERIOD 67 | -0.34 | PERIOD 92 | 0.12 |
| PERIOD 18 | 0.14 | PERIOD 43 | 0.05 | PERIOD 68 | 0.10 | PERIOD 93 | 0.10 |
| PERIOD 19 | 0.01 | PERIOD 44 | 0.05 | PERIOD 69 | 0.09 | PERIOD 94 | -0.25 |
| PERIOD 20 | 0.00 | PERIOD 45 | -0.03 | PERIOD 70 | 0.17 | PERIOD 95 | -0.09 |
| PERIOD 21 | -0.06 | PERIOD 46 | -0.06 | PERIOD 71 | 0.12 | PERIOD 96 | -0.05 |
| PERIOD 22 | 0.01 | PERIOD 47 | -0.07 | PERIOD 72 | -0.05 | PERIOD 97 | 0.07 |
| PERIOD 23 | 0.01 | PERIOD 48 | 0.05 | PERIOD 73 | -0.11 | PERIOD 98 | -0.01 |
| PERIOD 24 | -0.05 | PERIOD 49 | -0.04 | PERIOD 74 | 0.12 | PERIOD 99 | -0.01 |
| PERIOD 25 | 0.00 | PERIOD 50 | 0.08 | PERIOD 75 | 0.04 | PERIOD 100 | -0.03 |

Table 70. Combined Series with Target Region 11 Residuals continued

| PERIOD | RESID | PERIOD | RESID | PERIOD | RESID | PERIOD | RESID |
|---|---|---|---|---|---|---|---|
| PERIOD 101 | 0.15 | PERIOD 126 | -0.09 | PERIOD 151 | 0.06 | PERIOD 176 | 0.28 |
| PERIOD 102 | 0.09 | PERIOD 127 | -0.19 | PERIOD 152 | -0.19 | PERIOD 177 | 0.06 |
| PERIOD 103 | -0.17 | PERIOD 128 | 0.08 | PERIOD 153 | -0.05 | PERIOD 178 | -0.02 |
| PERIOD 104 | -0.05 | PERIOD 129 | 0.02 | PERIOD 154 | -0.04 | PERIOD 179 | 0.18 |
| PERIOD 105 | 0.03 | PERIOD 130 | 0.16 | PERIOD 155 | 0.08 | PERIOD 180 | -0.04 |
| PERIOD 106 | 0.02 | PERIOD 131 | -0.13 | PERIOD 156 | 0.01 | PERIOD 181 | 0.02 |
| PERIOD 107 | -0.05 | PERIOD 132 | -0.01 | PERIOD 157 | 0.16 | PERIOD 182 | 0.17 |
| PERIOD 108 | -0.05 | PERIOD 133 | -0.07 | PERIOD 158 | -0.03 | PERIOD 183 | -0.06 |
| PERIOD 109 | -0.20 | PERIOD 134 | 0.09 | PERIOD 159 | 0.00 | PERIOD 184 | 0.19 |
| PERIOD 110 | 0.04 | PERIOD 135 | -0.09 | PERIOD 160 | 0.01 | PERIOD 185 | -0.10 |
| PERIOD 111 | 0.06 | PERIOD 136 | -0.13 | PERIOD 161 | -0.09 | PERIOD 186 | -0.08 |
| PERIOD 112 | 0.27 | PERIOD 137 | -0.05 | PERIOD 162 | -0.07 | PERIOD 187 | -0.10 |
| PERIOD 113 | -0.02 | PERIOD 138 | 0.04 | PERIOD 163 | -0.01 | PERIOD 188 | -0.02 |
| PERIOD 114 | -0.02 | PERIOD 139 | 0.09 | PERIOD 164 | -0.03 | PERIOD 189 | -0.04 |
| PERIOD 115 | -0.11 | PERIOD 140 | 0.24 | PERIOD 165 | -0.03 | PERIOD 190 | 0.26 |
| PERIOD 116 | 0.02 | PERIOD 141 | 0.00 | PERIOD 166 | 0.41 | PERIOD 191 | 0.05 |
| PERIOD 117 | -0.08 | PERIOD 142 | -0.14 | PERIOD 167 | 0.09 | PERIOD 192 | -0.07 |
| PERIOD 118 | -0.12 | PERIOD 143 | -0.05 | PERIOD 168 | -0.01 | PERIOD 193 | 0.33 |
| PERIOD 119 | 0.04 | PERIOD 144 | 0.00 | PERIOD 169 | 0.46 | PERIOD 194 | -0.03 |
| PERIOD 120 | -0.03 | PERIOD 145 | -0.12 | PERIOD 170 | -0.07 | PERIOD 195 | -0.04 |
| PERIOD 121 | 0.04 | PERIOD 146 | 0.27 | PERIOD 171 | -0.10 | PERIOD 196 | 0.17 |
| PERIOD 122 | 0.01 | PERIOD 147 | -0.03 | PERIOD 172 | 0.05 | PERIOD 197 | -0.04 |
| PERIOD 123 | 0.01 | PERIOD 148 | -0.14 | PERIOD 173 | 0.02 | PERIOD 198 | -0.07 |
| PERIOD 124 | -0.19 | PERIOD 149 | -0.11 | PERIOD 174 | -0.08 | PERIOD 199 | 0.17 |
| PERIOD 125 | 0.06 | PERIOD 150 | 0.03 | PERIOD 175 | -0.36 | PERIOD 200 | 0.00 |
| | | | | | | PERIOD 201 | -0.04 |

## Appendix O. *Residual Autocorrelations and Partial Autocorrelations of the Combined Series with Target Region 11*

This appendix contains the residual autocorrelations and the residual partial autocorrelations from fitting the estimated SSTMA$(2_{1,1})_{12}$ model to the combined series with target region 7. The residual autocorrelations and residual autocorrelations were calculated for the first forty-four lags. Any residual autocorrelation or any residual partial autocorrelation that is greater than 0.14 or less than -0.14 is significantly different from zero. Each row represents a lag.

Table 71. Residual Autocorrelations and Residual Partial Autocorrelations of the Combined Series with Target Region 11

| | AC | PAC |
|---|---|---|
| LAG 1 | -0.03 | -0.03 |
| LAG 2 | -0.05 | -0.05 |
| LAG 3 | 0.02 | 0.02 |
| LAG 4 | -0.04 | -0.04 |
| LAG 5 | -0.07 | -0.07 |
| LAG 6 | -0.04 | -0.04 |
| LAG 7 | -0.09 | -0.10 |
| LAG 8 | -0.04 | -0.05 |
| LAG 9 | 0.15 | 0.14 |
| LAG 10 | 0.13 | 0.14 |
| LAG 11 | 0.09 | 0.11 |
| LAG 12 | -0.03 | -0.03 |
| LAG 13 | 0.05 | 0.04 |
| LAG 14 | 0.03 | 0.05 |
| LAG 15 | -0.07 | -0.04 |
| LAG 16 | -0.02 | 0.02 |
| LAG 17 | 0.02 | 0.06 |
| LAG 18 | 0.01 | 0.03 |
| LAG 19 | -0.01 | -0.04 |
| LAG 20 | 0.08 | 0.03 |
| LAG 21 | -0.01 | -0.01 |
| LAG 22 | -0.01 | -0.02 |

Table 72. Residual Autocorrelations and Residual Partial Autocorrelations of Combined Series with target Region 11 continued

| | AC | PAC |
|---|---|---|
| LAG 23 | 0.04 | 0.02 |
| LAG 24 | 0.04 | 0.05 |
| LAG 25 | -0.07 | -0.04 |
| LAG 26 | -0.01 | -0.01 |
| LAG 27 | 0.10 | 0.08 |
| LAG 28 | 0.04 | 0.07 |
| LAG 29 | 0.04 | 0.05 |
| LAG 30 | 0.05 | 0.05 |
| LAG 31 | -0.09 | -0.09 |
| LAG 32 | -0.03 | -0.03 |
| LAG 33 | 0.00 | -0.02 |
| LAG 34 | -0.05 | -0.03 |
| LAG 35 | -0.10 | -0.08 |
| LAG 36 | 0.01 | -0.01 |
| LAG 37 | -0.02 | -0.09 |
| LAG 38 | 0.01 | -0.06 |
| LAG 39 | 0.03 | -0.03 |
| LAG 40 | -0.05 | -0.08 |
| LAG 41 | 0.03 | 0.02 |
| LAG 42 | -0.01 | 0.00 |
| LAG 43 | -0.05 | -0.05 |
| LAG 44 | 0.03 | 0.04 |

# Appendix P. *Actual and Predicted Valued for $NHAS_{2,11,1}$*

This appendix contains actual and predicted values of $NHAS_{ijk}$ for event type 2, geographical region 11, and time block 1 for all sixty-seven observations. Each row represents a period.

Table 73. Actual and Predicted Values for $NHAS_{2,11,1}$

| | $NHAS_{2,11,1}$ | PREDICTED $NHAS_{2,11,1}$ |
|---|---|---|
| PERIOD 1 | 0.27 | 0.09 |
| PERIOD 2 | 0.00 | 0.09 |
| PERIOD 3 | 0.01 | 0.07 |
| PERIOD 4 | -0.01 | -0.03 |
| PERIOD 5 | -0.01 | 0.04 |
| PERIOD 6 | 0.00 | 0.05 |
| PERIOD 7 | -0.01 | -0.02 |
| PERIOD 8 | -0.01 | 0.01 |
| PERIOD 9 | 0.02 | 0.00 |
| PERIOD10 | 0.00 | 0.02 |
| PERIOD11 | -0.33 | -0.11 |
| PERIOD12 | 0.00 | -0.09 |
| PERIOD13 | -0.40 | -0.14 |
| PERIOD14 | 0.00 | -0.02 |
| PERIOD15 | -0.01 | -0.07 |
| PERIOD16 | 0.02 | 0.10 |
| PERIOD17 | 0.01 | 0.04 |
| PERIOD18 | 0.02 | 0.02 |
| PERIOD19 | 0.03 | 0.05 |
| PERIOD20 | 0.07 | 0.04 |
| PERIOD21 | 0.10 | 0.01 |
| PERIOD22 | 0.31 | 0.09 |
| PERIOD23 | -0.05 | 0.29 |
| PERIOD24 | 0.15 | -0.03 |
| PERIOD25 | 0.03 | 0.16 |
| PERIOD26 | -0.10 | 0.07 |
| PERIOD27 | -0.04 | -0.06 |
| PERIOD28 | 0.10 | 0.01 |
| PERIOD29 | -0.01 | 0.10 |
| PERIOD30 | 0.52 | 0.05 |
| PERIOD31 | 0.62 | 0.21 |
| PERIOD32 | 0.11 | 0.35 |

Table 74. Actual and Predicted Values for $NHAS_{2,11,1}$ continued

| | $NHAS_{2,11,1}$ | PREDICTED $NHAS_{2,11,1}$ |
|---|---|---|
| PERIOD33 | 0.09 | 0.04 |
| PERIOD34 | -0.23 | -0.21 |
| PERIOD35 | -0.03 | 0.14 |
| PERIOD36 | -0.12 | -0.11 |
| PERIOD37 | -0.13 | 0.06 |
| PERIOD38 | 0.31 | 0.03 |
| PERIOD39 | 0.01 | 0.14 |
| PERIOD40 | -0.04 | 0.07 |
| PERIOD41 | 0.03 | -0.02 |
| PERIOD42 | -0.46 | -0.26 |
| PERIOD43 | -0.55 | -0.35 |
| PERIOD44 | -0.02 | -0.19 |
| PERIOD45 | -0.12 | -0.04 |
| PERIOD46 | 0.00 | 0.12 |
| PERIOD47 | -0.04 | 0.04 |
| PERIOD48 | -0.16 | -0.01 |
| PERIOD49 | -0.04 | 0.09 |
| PERIOD50 | -0.33 | -0.19 |
| PERIOD51 | 0.02 | -0.03 |
| PERIOD52 | -0.02 | 0.03 |
| PERIOD53 | 0.22 | 0.05 |
| PERIOD54 | 0.21 | 0.21 |
| PERIOD55 | 0.24 | 0.25 |
| PERIOD56 | 0.45 | 0.02 |
| PERIOD57 | 0.71 | 0.26 |
| PERIOD58 | 0.50 | 0.46 |
| PERIOD59 | -0.01 | 0.33 |
| PERIOD60 | 0.01 | 0.03 |
| PERIOD61 | 0.02 | 0.03 |
| PERIOD62 | 0.38 | 0.17 |
| PERIOD63 | 0.07 | 0.16 |
| PERIOD64 | 0.38 | 0.12 |
| PERIOD65 | 0.34 | 0.02 |
| PERIOD66 | 0.42 | 0.24 |
| PERIOD67 | 0.37 | 0.20 |

## Appendix Q. *Actual, Predicted, and Transformed Predicted Values of the Historical Frequencies for Event Type 2, Geographical Region 11, and Time Block 1*

This appendix contains the actual, predicted, and transformed predicted values of the monthly observations of the observed historical frequency $X_{ijk}$ for event type 2, geographical region 11, and time block 1. Each row represents a month.

Table 75. Actual, Predicted, and Transformed Predicted Values of Historical Frequencies $X_{2,11,1}$

| | PREDICTED $X_{2,11,1}$ | TRANSFORMED $X_{2,11,1}$ | ACTUAL $X_{2,11,1}$ |
|---|---|---|---|
| JAN 1985 | 0.13 | 0.13 | 0.13 |
| FEB 1985 | 0.00 | 0.00 | 0.00 |
| MAR 1985 | 0.00 | 0.00 | 0.00 |
| APR 1985 | 0.00 | 0.00 | 0.00 |
| MAY 1985 | 0.00 | 0.00 | 0.00 |
| JUN 1985 | 0.00 | 0.00 | 0.00 |
| JUL 1985 | 0.00 | 0.00 | 0.00 |
| AUG 1985 | 0.00 | 0.00 | 0.00 |
| SEP 1985 | 0.00 | 0.00 | 0.00 |
| OCT 1985 | 0.00 | 0.00 | 0.00 |
| NOV 1985 | 0.33 | 0.33 | 0.33 |
| DEC 1985 | 0.00 | 0.00 | 0.00 |
| JAN 1986 | 0.22 | 0.22 | 0.40 |
| FEB 1986 | 0.09 | 0.09 | 0.00 |
| MAR 1986 | 0.06 | 0.06 | 0.00 |
| APR 1986 | -0.02 | 0.00 | 0.00 |
| MAY 1986 | 0.05 | 0.05 | 0.00 |
| JUN 1986 | 0.05 | 0.05 | 0.00 |
| JUL 1986 | -0.01 | 0.00 | 0.00 |
| AUG 1986 | 0.02 | 0.02 | 0.00 |
| SEP 1986 | -0.02 | 0.00 | 0.00 |
| OCT 1986 | 0.02 | 0.02 | 0.00 |
| NOV 1986 | 0.22 | 0.22 | 0.00 |
| DEC 1986 | -0.09 | 0.00 | 0.00 |
| JAN 1987 | 0.26 | 0.26 | 0.00 |
| FEB 1987 | -0.02 | 0.00 | 0.00 |
| MAR 1987 | -0.05 | 0.00 | 0.01 |

Table 76. Actual, Predicted, and Transformed Predicted Values of Historical Frequencies $X_{2,11,1}$ continued

| | PREDICTED $X_{2,11,1}$ | TRANSFORMED $X_{2,11,1}$ | ACTUAL $X_{2,11,1}$ |
|---|---|---|---|
| APR 1987 | 0.08 | 0.08 | 0.00 |
| MAY 1987 | 0.03 | 0.03 | 0.00 |
| JUN 1987 | 0.00 | 0.00 | 0.00 |
| JUL 1987 | 0.02 | 0.02 | 0.00 |
| AUG 1987 | 0.00 | 0.00 | 0.03 |
| SEP 1987 | 0.02 | 0.02 | 0.11 |
| OCT 1987 | 0.09 | 0.09 | 0.31 |
| NOV 1987 | 0.39 | 0.39 | 0.05 |
| DEC 1987 | -0.01 | 0.00 | 0.17 |
| JAN 1988 | 0.20 | 0.20 | 0.07 |
| FEB 1988 | 0.17 | 0.17 | 0.00 |
| MAR 1988 | -0.02 | 0.00 | 0.00 |
| APR 1988 | -0.01 | 0.00 | 0.08 |
| MAY 1988 | 0.11 | 0.11 | 0.00 |
| JUN 1988 | 0.03 | 0.03 | 0.50 |
| JUL 1988 | 0.17 | 0.17 | 0.58 |
| AUG 1988 | 0.35 | 0.35 | 0.11 |
| SEP 1988 | 0.11 | 0.11 | 0.16 |
| OCT 1988 | 0.06 | 0.06 | 0.04 |
| NOV 1988 | 0.22 | 0.22 | 0.05 |
| DEC 1988 | 0.16 | 0.16 | 0.15 |
| JAN 1989 | 0.22 | 0.22 | 0.03 |
| FEB 1989 | 0.05 | 0.05 | 0.33 |
| MAR 1989 | 0.13 | 0.13 | 0.00 |
| APR 1989 | 0.13 | 0.13 | 0.02 |

Table 77. Actual, Predicted, and Transformed Predicted Values of Historical Frequencies $X_{2,11,1}$ continued

| | PREDICTED $X_{2,11,1}$ | TRANSFORMED $X_{2,11,1}$ | ACTUAL $X_{2,11,1}$ |
|---|---|---|---|
| MAY 1989 | -0.04 | 0.00 | 0.01 |
| JUN 1989 | 0.22 | 0.22 | 0.02 |
| JUL 1989 | 0.21 | 0.21 | 0.01 |
| AUG 1989 | -0.10 | 0.00 | 0.07 |
| SEP 1989 | 0.10 | 0.10 | 0.02 |
| OCT 1989 | 0.16 | 0.16 | 0.04 |
| NOV 1989 | 0.11 | 0.11 | 0.03 |
| DEC 1989 | 0.15 | 0.15 | 0.00 |
| JAN 1990 | 0.13 | 0.13 | 0.00 |
| FEB 1990 | 0.14 | 0.14 | 0.00 |
| MAR 1990 | -0.03 | 0.00 | 0.02 |
| APR 1990 | 0.05 | 0.05 | 0.00 |
| MAY 1990 | 0.06 | 0.06 | 0.23 |
| JUN 1990 | 0.24 | 0.24 | 0.24 |
| JUL 1990 | 0.27 | 0.27 | 0.26 |
| AUG 1990 | 0.10 | 0.10 | 0.53 |
| SEP 1990 | 0.29 | 0.29 | 0.74 |
| OCT 1990 | 0.50 | 0.50 | 0.54 |
| NOV 1990 | 0.34 | 0.34 | 0.00 |
| DEC 1990 | 0.02 | 0.02 | 0.00 |
| JAN 1991 | 0.03 | 0.03 | 0.02 |
| FEB 1991 | 0.17 | 0.17 | 0.38 |
| MAR 1991 | 0.17 | 0.17 | 0.08 |
| APR 1991 | 0.12 | 0.12 | 0.38 |
| MAY 1991 | 0.25 | 0.25 | 0.57 |
| JUN 1991 | 0.48 | 0.48 | 0.66 |
| JUL 1991 | 0.47 | 0.47 | 0.64 |

## Appendix R. *Combined Series With Target Region 7*

This appendix contains the the $NHAS_{2,j,1}$ values for target region 7, the weighted sum of the first order neighbors of target region 7, and the weighted sum of the second order neighbors of target region 7. The combined series results if the rows are read across. Each row represents a period.

Table 78. Combined Series for Target Region 7

| | TARGET 7 | 1ST ORDER | 2ND ORDER |
|---|---|---|---|
| PERIOD 1 | 0.27 | 0.17 | -0.11 |
| PERIOD 2 | 0.00 | 0.02 | -0.10 |
| PERIOD 3 | 0.01 | -0.03 | 0.11 |
| PERIOD 4 | -0.01 | -0.18 | -0.05 |
| PERIOD 5 | -0.01 | -0.06 | -0.06 |
| PERIOD 6 | 0.00 | -0.06 | 0.10 |
| PERIOD 7 | -0.01 | 0.00 | 0.00 |
| PERIOD 8 | -0.01 | 0.00 | 0.00 |
| PERIOD 9 | 0.02 | 0.00 | -0.16 |
| PERIOD 10 | 0.00 | -0.07 | -0.07 |
| PERIOD 11 | -0.33 | -0.36 | 0.00 |
| PERIOD 12 | 0.00 | 0.10 | -0.08 |
| PERIOD 13 | -0.40 | -0.25 | -0.03 |
| PERIOD 14 | 0.00 | -0.01 | 0.11 |
| PERIOD 15 | -0.01 | 0.00 | -0.08 |
| PERIOD 16 | 0.02 | 0.02 | 0.12 |
| PERIOD 17 | 0.01 | 0.11 | 0.00 |
| PERIOD 18 | 0.02 | 0.03 | -0.06 |
| PERIOD 19 | 0.03 | 0.01 | 0.00 |
| PERIOD 20 | 0.07 | 0.20 | 0.05 |
| PERIOD 21 | 0.10 | 0.08 | 0.07 |
| PERIOD 22 | 0.31 | 0.18 | 0.06 |
| PERIOD 23 | -0.05 | 0.00 | 0.07 |
| PERIOD 24 | 0.15 | 0.01 | -0.01 |
| PERIOD 25 | 0.03 | 0.02 | 0.08 |
| PERIOD 26 | -0.10 | -0.02 | -0.06 |
| PERIOD 27 | -0.04 | 0.07 | -0.08 |
| PERIOD 28 | 0.10 | 0.04 | -0.04 |
| PERIOD 29 | -0.01 | -0.08 | -0.04 |
| PERIOD 30 | 0.52 | 0.22 | -0.07 |
| PERIOD 31 | 0.62 | 0.27 | 0.04 |
| PERIOD 32 | 0.11 | -0.05 | -0.02 |

Table 79. Combined Series for Target Region 7 continued

| | TARGET 7 | 1ST ORDER | 2ND ORDER |
|---|---|---|---|
| PERIOD 33 | 0.09 | 0.07 | 0.08 |
| PERIOD 34 | -0.23 | 0.06 | 0.09 |
| PERIOD 35 | -0.03 | 0.05 | 0.04 |
| PERIOD 36 | -0.12 | -0.07 | 0.07 |
| PERIOD 37 | -0.13 | -0.10 | 0.06 |
| PERIOD 38 | 0.31 | 0.05 | 0.03 |
| PERIOD 39 | 0.01 | 0.02 | 0.03 |
| PERIOD 40 | -0.04 | 0.11 | -0.04 |
| PERIOD 41 | 0.03 | 0.09 | 0.04 |
| PERIOD 42 | -0.46 | -0.10 | -0.01 |
| PERIOD 43 | -0.55 | -0.11 | 0.00 |
| PERIOD 44 | -0.02 | -0.02 | 0.03 |
| PERIOD 45 | -0.12 | 0.04 | -0.06 |
| PERIOD 46 | 0.00 | -0.16 | 0.00 |
| PERIOD 47 | -0.04 | -0.03 | -0.05 |
| PERIOD 48 | -0.16 | -0.17 | 0.01 |
| PERIOD 49 | -0.04 | -0.04 | -0.07 |
| PERIOD 50 | -0.33 | -0.06 | 0.04 |
| PERIOD 51 | 0.02 | 0.01 | 0.06 |
| PERIOD 52 | -0.02 | -0.03 | 0.08 |
| PERIOD 53 | 0.22 | 0.04 | 0.03 |
| PERIOD 54 | 0.21 | 0.01 | 0.04 |
| PERIOD 55 | 0.24 | 0.02 | 0.00 |
| PERIOD 56 | 0.45 | 0.18 | -0.02 |
| PERIOD 57 | 0.71 | 0.15 | -0.04 |
| PERIOD 58 | 0.50 | 0.21 | -0.09 |
| PERIOD 59 | -0.01 | -0.07 | 0.00 |
| PERIOD 60 | 0.01 | 0.00 | -0.08 |
| PERIOD 61 | 0.02 | 0.03 | -0.03 |
| PERIOD 62 | 0.38 | 0.06 | -0.10 |
| PERIOD 63 | 0.07 | 0.04 | -0.05 |
| PERIOD 64 | 0.38 | 0.07 | -0.10 |
| PERIOD 65 | 0.34 | 0.12 | -0.05 |
| PERIOD 66 | 0.42 | 0.15 | -0.05 |
| PERIOD 67 | 0.37 | 0.14 | -0.03 |

## Appendix S. *Residual Autocorrelations and Partial Autocorrelations for Combined Series with Target Region 7*

This appendix contains the residual autocorrelations and the residual partial autocorrelations from fitting the estimated SSTMA$(2_{1,1})_{12}$ model created on target region 11 to the combined series with target region 7. The residual autocorrelations and residual autocorrelations were calculated for the first forty-four lags. Any residual autocorrelation or any residual partial autocorrelation that is greater than 0.14 or less than -0.14 is significantly different from zero. Each row represents a lag.

Table 80. Residual Autocorrelations and Residual Partial Autocorrelations for Combined Series with Target Region 7

| | AC | PAC |
|---|---|---|
| LAG 1 | -0.03 | -0.03 |
| LAG 2 | -0.05 | -0.05 |
| LAG 3 | 0.02 | 0.02 |
| LAG 4 | -0.04 | -0.04 |
| LAG 5 | -0.07 | -0.07 |
| LAG 6 | -0.04 | -0.04 |
| LAG 7 | -0.09 | -0.10 |
| LAG 8 | -0.04 | -0.05 |
| LAG 9 | 0.15 | 0.14 |
| LAG 10 | 0.13 | 0.14 |
| LAG 11 | 0.09 | 0.11 |
| LAG 12 | -0.03 | -0.03 |
| LAG 13 | 0.05 | 0.04 |
| LAG 14 | 0.03 | 0.05 |
| LAG 15 | -0.07 | -0.04 |
| LAG 16 | -0.02 | 0.02 |
| LAG 17 | 0.02 | 0.06 |
| LAG 18 | 0.01 | 0.03 |
| LAG 19 | -0.01 | -0.04 |
| LAG 20 | 0.08 | 0.03 |
| LAG 21 | -0.01 | -0.01 |
| LAG 22 | -0.01 | -0.02 |

Table 81. Residual Autocorrelations and Residual Partial Autocorrelations for Combined Series with Target Region 7 continued

| | AC | PAC |
|---|---|---|
| LAG 23 | 0.04 | 0.02 |
| LAG 24 | 0.04 | 0.05 |
| LAG 25 | -0.07 | -0.04 |
| LAG 26 | -0.01 | -0.01 |
| LAG 27 | 0.10 | 0.08 |
| LAG 28 | 0.04 | 0.07 |
| LAG 29 | 0.04 | 0.05 |
| LAG 30 | 0.05 | 0.05 |
| LAG 31 | -0.09 | -0.09 |
| LAG 32 | -0.03 | -0.03 |
| LAG 33 | 0.00 | -0.02 |
| LAG 34 | -0.05 | -0.03 |
| LAG 35 | -0.10 | -0.08 |
| LAG 36 | 0.01 | -0.01 |
| LAG 37 | -0.02 | -0.09 |
| LAG 38 | 0.01 | -0.06 |
| LAG 39 | 0.03 | -0.03 |
| LAG 40 | -0.05 | -0.08 |
| LAG 41 | 0.03 | 0.02 |
| LAG 42 | -0.01 | 0.00 |
| LAG 43 | -0.05 | -0.05 |
| LAG 44 | 0.03 | 0.04 |

## Appendix T. *Actual and Predicted Valued for* $NHAS_{2,7,1}$

This appendix contains actual and predicted values of $NHAS_{ijk}$ for event type 2, geographical region 7, and time block 1 for all sixty-seven observations. Each row represents a period.

Table 82. Actual and Predicted Values for $NHAS_{2,7,1}$

| | PREDICTED $NHAS_{2,7,1}$ | $NHAS_{2,7,1}$ |
|---|---|---|
| PERIOD 1 | 0.09 | 0.27 |
| PERIOD 2 | 0.09 | 0.00 |
| PERIOD 3 | 0.09 | 0.01 |
| PERIOD 4 | -0.03 | -0.01 |
| PERIOD 5 | 0.03 | -0.01 |
| PERIOD 6 | 0.08 | 0.00 |
| PERIOD 7 | -0.02 | -0.01 |
| PERIOD 8 | -0.02 | -0.01 |
| PERIOD 9 | 0.02 | 0.02 |
| PERIOD 10 | 0.03 | 0.00 |
| PERIOD 11 | -0.11 | -0.33 |
| PERIOD 12 | -0.09 | 0.00 |
| PERIOD 13 | -0.15 | -0.40 |
| PERIOD 14 | 0.00 | 0.00 |
| PERIOD 15 | -0.06 | -0.01 |
| PERIOD 16 | 0.08 | 0.02 |
| PERIOD 17 | 0.05 | 0.01 |
| PERIOD 18 | 0.02 | 0.02 |
| PERIOD 19 | 0.05 | 0.03 |
| PERIOD 20 | 0.04 | 0.07 |
| PERIOD 21 | 0.01 | 0.10 |
| PERIOD 22 | 0.07 | 0.31 |
| PERIOD 23 | 0.29 | -0.05 |
| PERIOD 24 | -0.02 | 0.15 |
| PERIOD 25 | 0.14 | 0.03 |
| PERIOD 26 | 0.08 | -0.10 |
| PERIOD 27 | -0.06 | -0.04 |
| PERIOD 28 | 0.01 | 0.10 |
| PERIOD 29 | 0.11 | -0.01 |
| PERIOD 30 | 0.05 | 0.52 |
| PERIOD 31 | 0.22 | 0.62 |
| PERIOD 32 | 0.36 | 0.11 |
| PERIOD 33 | 0.02 | 0.09 |

Table 83. Actual and Predicted Values for $NHAS_{2,7,1}$ continued

| | PREDICTED $NHAS_{2,7,1}$ | $NHAS_{2,7,1}$ |
|---|---|---|
| PERIOD 34 | -0.20 | -0.23 |
| PERIOD 35 | 0.15 | -0.03 |
| PERIOD 36 | -0.14 | -0.12 |
| PERIOD 37 | 0.07 | -0.13 |
| PERIOD 38 | 0.04 | 0.31 |
| PERIOD 39 | 0.12 | 0.01 |
| PERIOD 40 | 0.08 | -0.04 |
| PERIOD 41 | -0.01 | 0.03 |
| PERIOD 42 | -0.27 | -0.46 |
| PERIOD 43 | -0.36 | -0.55 |
| PERIOD 44 | -0.18 | -0.02 |
| PERIOD 45 | -0.05 | -0.12 |
| PERIOD 46 | 0.13 | 0.00 |
| PERIOD 47 | 0.05 | -0.04 |
| PERIOD 48 | -0.02 | -0.16 |
| PERIOD 49 | 0.08 | -0.04 |
| PERIOD 50 | -0.19 | -0.33 |
| PERIOD 51 | -0.04 | 0.02 |
| PERIOD 52 | 0.02 | -0.02 |
| PERIOD 53 | 0.06 | 0.22 |
| PERIOD 54 | 0.20 | 0.21 |
| PERIOD 55 | 0.25 | 0.24 |
| PERIOD 56 | 0.04 | 0.45 |
| PERIOD 57 | 0.25 | 0.71 |
| PERIOD 58 | 0.48 | 0.50 |
| PERIOD 59 | 0.35 | -0.01 |
| PERIOD 60 | 0.03 | 0.01 |
| PERIOD 61 | 0.00 | 0.02 |
| PERIOD 62 | 0.19 | 0.38 |
| PERIOD 63 | 0.17 | 0.07 |
| PERIOD 64 | 0.12 | 0.38 |
| PERIOD 65 | 0.01 | 0.34 |
| PERIOD 66 | 0.25 | 0.42 |
| PERIOD 67 | 0.20 | 0.37 |

## Appendix U. *Actual, Predicted, and Transformed Predicted Values of the Historical Frequencies for Event Type 2, Geographical Region 7, and Time Block 1*

This appendix contains the actual, predicted, and transformed predicted values of the monthly observations of the observed historical frequency $X_{ijk}$ for event type 2, geographical region 7, and time block 1. Each row represents a month.

Table 84. Actual, Predicted, and Transformed Predicted Values of Historical Frequencies $X_{2,7,1}$

| | PREDICTED $X_{2,7,1}$ | TRANSFORMED $X_{2,7,1}$ | ACTUAL $X_{2,7,1}$ |
|---|---|---|---|
| JAN 1985 | 0.00 | 0.00 | 0.00 |
| FEB 1985 | 0.00 | 0.00 | 0.00 |
| MAR 1985 | 0.33 | 0.33 | 0.33 |
| APR 1985 | 0.00 | 0.00 | 0.00 |
| MAY 1985 | 0.00 | 0.00 | 0.00 |
| JUN 1985 | 0.00 | 0.00 | 0.00 |
| JUL 1985 | 0.00 | 0.00 | 0.00 |
| AUG 1985 | 0.00 | 0.00 | 0.00 |
| SEP 1985 | 0.00 | 0.00 | 0.00 |
| OCT 1985 | 0.18 | 0.18 | 0.18 |
| NOV 1985 | 0.00 | 0.00 | 0.00 |
| DEC 1985 | 0.00 | 0.00 | 0.00 |
| JAN 1986 | 0.19 | 0.19 | 0.00 |
| FEB 1986 | 0.08 | 0.08 | 0.00 |
| MAR 1986 | 0.43 | 0.43 | 0.00 |
| APR 1986 | -0.02 | 0.00 | 0.00 |
| MAY 1986 | 0.03 | 0.03 | 0.50 |
| JUN 1986 | 0.08 | 0.08 | 0.00 |
| JUL 1986 | -0.02 | 0.00 | 0.00 |
| AUG 1986 | -0.02 | 0.00 | 0.00 |
| SEP 1986 | 0.04 | 0.04 | 0.00 |
| OCT 1986 | 0.20 | 0.20 | 0.00 |
| NOV 1986 | -0.09 | 0.00 | 0.00 |
| DEC 1986 | 0.01 | 0.01 | 0.00 |
| JAN 1987 | -0.19 | 0.00 | 0.00 |
| FEB 1987 | 0.01 | 0.01 | 0.06 |
| MAR 1987 | -0.07 | 0.00 | 0.30 |

Table 85. Actual, Predicted, and Transformed Predicted Values of Historical Frequencies $X_{2,7,1}$ continued

| | PREDICTED $X_{2,7,1}$ | TRANSFORMED $X_{2,7,1}$ | ACTUAL $X_{2,7,1}$ |
|---|---|---|---|
| APR 1987 | 0.06 | 0.06 | 0.04 |
| MAY 1987 | 0.54 | 0.54 | 0.00 |
| JUN 1987 | 0.02 | 0.02 | 0.00 |
| JUL 1987 | 0.04 | 0.04 | 0.00 |
| AUG 1987 | 0.02 | 0.02 | 0.06 |
| SEP 1987 | -0.04 | 0.00 | 0.43 |
| OCT 1987 | 0.06 | 0.06 | 0.03 |
| NOV 1987 | 0.21 | 0.21 | 0.30 |
| DEC 1987 | -0.19 | 0.00 | 0.17 |
| JAN 1988 | -0.03 | 0.00 | 0.24 |
| FEB 1988 | 0.05 | 0.05 | 0.50 |
| MAR 1988 | 0.24 | 0.24 | 0.67 |
| APR 1988 | 0.05 | 0.05 | 0.08 |
| MAY 1988 | 0.10 | 0.10 | 0.67 |
| JUN 1988 | 0.05 | 0.05 | 0.00 |
| JUL 1988 | 0.21 | 0.21 | 0.04 |
| AUG 1988 | 0.41 | 0.41 | 0.06 |
| SEP 1988 | 0.42 | 0.42 | 0.18 |
| OCT 1988 | -0.21 | 0.00 | 0.04 |
| NOV 1988 | 0.35 | 0.35 | 0.14 |
| DEC 1988 | -0.10 | 0.00 | 0.00 |
| JAN 1989 | 0.21 | 0.21 | 0.07 |
| FEB 1989 | 0.52 | 0.52 | 0.20 |
| MAR 1989 | 0.74 | 0.74 | 0.38 |
| APR 1989 | 0.14 | 0.14 | 0.10 |

Table 86. Actual, Predicted, and Transformed Predicted Values of Historical Frequencies $X_{2,7,1}$ continued

| | PREDICTED $X_{2,7,1}$ | TRANSFORMED $X_{2,7,1}$ | ACTUAL $X_{2,7,1}$ |
|---|---|---|---|
| MAY 1989 | 0.66 | 0.66 | 0.00 |
| JUN 1989 | -0.27 | 0.00 | 0.02 |
| JUL 1989 | -0.32 | 0.00 | 0.02 |
| AUG 1989 | -0.12 | 0.00 | 0.00 |
| SEP 1989 | 0.12 | 0.12 | 0.16 |
| OCT 1989 | 0.17 | 0.17 | 0.20 |
| NOV 1989 | 0.19 | 0.19 | 0.57 |
| DEC 1989 | -0.03 | 0.00 | 0.20 |
| JAN 1990 | 0.14 | 0.14 | 0.65 |
| FEB 1990 | 0.01 | 0.01 | 0.41 |
| MAR 1990 | 0.34 | 0.34 | 0.09 |
| APR 1990 | 0.12 | 0.12 | 0.01 |
| MAY 1990 | 0.06 | 0.06 | 0.00 |
| JUN 1990 | 0.22 | 0.22 | 0.00 |
| JUL 1990 | 0.27 | 0.27 | 0.00 |
| AUG 1990 | 0.04 | 0.04 | 0.05 |
| SEP 1990 | 0.41 | 0.41 | 0.00 |
| OCT 1990 | 0.68 | 0.68 | 0.14 |
| NOV 1990 | 0.91 | 0.91 | 0.87 |
| DEC 1990 | 0.24 | 0.24 | 0.70 |
| JAN 1991 | 0.67 | 0.67 | 0.88 |
| FEB 1991 | 0.61 | 0.61 | 0.46 |
| MAR 1991 | 0.27 | 0.27 | 0.27 |
| APR 1991 | 0.13 | 0.13 | 0.06 |
| MAY 1991 | 0.01 | 0.01 | 0.01 |
| JUN 1991 | 0.25 | 0.25 | 0.00 |
| JUL 1991 | 0.20 | 0.20 | 0.00 |

## Appendix V. *Description of Floppy Disk Files*

All of the floppy disk files are in *LOTUS 1-2-3* format. The files can be loaded into either *LOTUS 1-2-3* or *QuattroPro*.

HISTORICAL.WKE: Monthly historical relative frequencies $X_{2,j,1}$ for all twenty-two geographical regions from January of 1985 through July of 1991.

ANALYTICAL.WKE: Monthly analytical predictions $\breve{p}_{2,j,1}$ for all twenty-two geographical regions from January of 1985 through July of 1991.

NORMAN.WKE: Normalized monthly analytical predictions for all twenty-two geographical regions from January of 1985 through July of 1991.

NHA.WKE: Database containing observations of normalized monthly analytical predictions subtracted from monthly historical relative frequencies $NHA_{2,j,1}$ for all twenty-two geographical regions from January of 1985 through July of 1991.

ACNHA.WKE: Autocorrelations of $NHA_{2,j,1}$ for all twenty-two geographical regions calculated for the first twenty lags.

NHAS.WKE: Database containing the de-seasonalized $NHA_{2,j,1}$ values $NHAS_{2,j,1}$ for all twenty-two geographical regions from period 1 though period 67.

ACNHAS.WKE: Autocorrelations of $NHAS_{2,j,1}$ for all twenty-two geographical regions calculated for the first seventeen lags.

PACNHAS1.WKE: Partial autocorrelations of $NHAS_{2,11,1}$ calculated for the first seventeen lags.

SERIES11.WKE: Combined series with target region 11.

ACCOMBIN.WKE: Autocorrelations and partial autocorrelations for the combined series with target region 11 calculated for the first forty-four lags.

ERROR21.WKE: Residuals from fitting the estimated SSTMA$(2_{1,1})_{12}$ model to the combined series with target region 11.

PACERROR.WKE: Residual autocorrelations and residual partial autocorrelations from fitting the estimated SSTMA$(2_{1,1})_{12}$ model to the combined series with target region 11 calculated for the first forty-four lags.

FITNHAS1.WKE: Fitted values of $NHAS_{2,11,1}$ using the estimated SSTMA$(2_{1,1})_{12}$ model.

FITTGT11.WKE: Predictions $\hat{p}_{2,11,1}$ using the estimated SSTMA$(2_{1,1})_{12}$ model.

STARMA7.WKE: Combined series with target region 7.

ACRESID7.WKE: Residual autocorrelations and residual partial autocorrelations from fitting the combined series with target region 7 with the estimated SSTMA$(2_{1,1})_{12}$ model developed from the combined series with target region 11 calculated for the first forty-four lags.

FITNHAS7.WKE: Fitted values of $NHAS_{2,7,1}$ using the estimated SSTMA$(2_{1,1})_{12}$ model developed from the combined series with target region 11.

FITTGT7.WKE: Predictions $\hat{p}_{2,7,1}$ using the estimated SSTMA$(2_{1,1})_{12}$ model developed from the combined series with target region 11.

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## *Vita*

Second Lieutenant Kelly A Greene was born Kelly A Kratochvil on 2 October 1968 in Oakland, California. In 1986, she graduated from Glen A Wilson High School in Hacienda Heights, California, and attended the United States Air Force Academy. In May of 1990, she graduated from the Academy with a Bachelor of Science in Operations Research. She entered the School of Engineering, Air Force Institute of Technology in August of 1990.

Permanent address: 320 Waterfront Place
Centerville, OH 45458-3868

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**13. ABSTRACT** *(Maximum 200 words)*

The Department of Defense employs a resource limited world-wide sensor system to detect certain "events" of interest. The purpose of this research was to establish a methodolgy using a univariate causal STARMA model for forecasting the relative probability of an event occurring in a geographical location during a time block of the day. These relative probabilities are used as input for a tasking model that assigns the scarce sensor resources so as to optimize the detection of these events. The STARMA model is appropriate for forecasting the relative probabilities because a definte temporal relationship and a definite spatial relationship exists in the data bases. The model created is a univariate causal STARMA model in that it only produces forecasts for one of the twenty-two given geographical regions. A causal univariate STARMA model was created to provide forecasts for one event type occurring at region 11 and appears to provide good forecasts. The model is both correlative and causal. The model is correlative in that it uses temporal and spatial correlations to develop the forecasts. The model is also causal in that it employs predictions from an analytical model.

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